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Muhammad Akram · Fariha Zafar

Hybrid Soft Computing Models Applied to Graph Theory

 Springer

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We dedicate this book to Prof. Lotfi A. Zadeh!

Preface

Fuzzy set theory was introduced by Lotfi Zadeh in 1965, as a generalization of classical set theory, for representing imprecise and vague phenomena. The concept of fuzzy graphs was initiated by Kaufmann based on Zadeh's fuzzy relations. In 1975, Rosenfeld laid the foundations for fuzzy graph theory. Professors Mordeson and Nair made a real contribution in putting together a very comprehensive book on 'Fuzzy Graphs and Fuzzy Hypergraphs', which motivated us to work in this direction.

Due to recent advances in science and technology, traditional mathematical tools are not sufficient for dealing with the complex problems arising in our real world day by day. To address these increasing challenges, there is a need for novel and innovative mathematical tools. The biggest dilemma of our universe is uncertainty, and the traditional crisp methods fail to handle these uncertainties in some complex problems. Many researchers extended the classical sets to various new models like fuzzy sets, soft sets, intuitionistic fuzzy sets, rough sets, bipolar fuzzy sets and many others to address the problems related to vagueness and uncertainty. Due to the limitation of human's knowledge to understand the complex problems, it is very difficult to apply only a single type of uncertainty method to deal with such problems. Therefore, it is necessary to develop hybrid models by incorporating the advantages of many other different mathematical models dealing with uncertainty. Hybrid computational intelligence is an effective combination of intelligent techniques that performs superior or in a competitive way to simple standard intelligent techniques. The increased popularity of hybrid intelligent systems in recent times lies in the extensive success of these systems in many real-world complex problems. The work presented here intends to deal with different sets of data and complex problems through hybrid models. This monograph deals with some hybrid models and their applications. It is based on a number of papers by the authors, which have been published in various scientific journals. The book may be useful for researchers in mathematics, computer scientists and social scientists alike.

In Chap. 1, several basic notions concerning rough fuzzy digraphs are discussed. Different methods of construction of rough fuzzy digraphs are described. Applications of rough fuzzy digraphs in decision-making problems are presented.

In Chap. 2, the concept of fuzzy rough sets is applied to graphs. Some applications of fuzzy rough digraphs are presented. Moreover, a comparative study of fuzzy rough digraphs with rough fuzzy digraphs and fuzzy digraphs is done.

In Chap. 3, an intuitionistic fuzzy rough model is presented. Some operations and products of intuitionistic fuzzy rough graphs are discussed in detail. Some efficient algorithms are developed to solve decision-making problems.

In Chap. 4, the concept of fuzzy soft graphs is presented. Some notions, including strong fuzzy soft graphs, complete fuzzy soft graphs, regular fuzzy soft graphs, fuzzy soft trees, fuzzy soft cycles, fuzzy soft bridges and fuzzy soft cutnodes are discussed. Applications of fuzzy soft graphs in decision-making problems are also presented.

In Chap. 5, the concept of intuitionistic fuzzy soft graphs is presented. Some notions of possibility intuitionistic fuzzy soft graph, regular, irregular, edge regular, edge irregular and strongly edge irregular intuitionistic fuzzy soft graphs are also presented. Intuitionistic fuzzy soft graphs are applied to multi-attribute decision-making problems.

In Chap. 6, the notions of soft rough digraphs and soft rough fuzzy digraphs are presented. Soft rough fuzzy model is applied to describe and resolve some multi-criteria decision-making problems.

In Chap. 7, certain notions of bipolar fuzzy soft graphs are presented. Some of their properties are also investigated. Several applications of the bipolar fuzzy soft graphs in a multiple criteria decision-making problem are presented.

In Chap. 8, certain concepts including soft rough neutrosophic graphs, soft rough neutrosophic influence graphs, soft rough neutrosophic influence cycles and soft rough neutrosophic influence trees are presented. A decision-making problem is solved by using the proposed algorithm.

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Lahore, Pakistan

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