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Rafael G. Campos

The XFT Quadrature in Discrete Fourier Analysis

Birkhäuser

Rafael G. Campos
Science Department
University of Quintana Roo
Chetumal, Mexico

ISSN 2296-5009 ISSN 2296-5017 (electronic)
Applied and Numerical Harmonic Analysis
ISBN 978-3-030-13422-8 ISBN 978-3-030-13423-5 (eBook)
https://doi.org/10.1007/978-3-030-13423-5

Library of Congress Control Number: 2019934448

Mathematics Subject Classification (2010): 14K25, 26A33, 26A36, 33C45, 33C47, 33E05, 34A08, 35R11, 41A05, 41A55, 42A15, 42A38, 42A85, 42C10, 44A20, 44A35, 44A55, 65D25, 65D30, 65D32, 65L05, 65L20, 65R10, 65T40, 65T50

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The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

ANHA Series Preface

The *Applied and Numerical Harmonic Analysis (ANHA)* book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art *ANHA* series.

Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlying mathematics of wavelet theory depends not only on classical Fourier analysis, but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods. The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key role of *ANHA*. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

Antenna theory	*Prediction theory*
Biomedical signal processing	*Radar applications*
Digital signal processing	*Sampling theory*
Fast algorithms	*Spectral estimation*
Gabor theory and applications	*Speech processing*
Image processing	*Time-frequency and*
Numerical partial differential equations	*time-scale analysis*
	Wavelet theory

The above point of view for the *ANHA* book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of “function.” Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor’s set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener’s Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptive modeling inherent in time-frequency-scale methods such as wavelet theory. The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory. We are back to the raison d'être of the *ANHA* series!

University of Maryland
College Park

John J. Benedetto
Series Editor

Preface

The main purpose of this book is to present, in a structured way, some new results on discrete and numerical Fourier analysis which are beyond the standard discrete Fourier transform and to put them in the context of the application-oriented Fourier analysis. This book has two objectives: the first one is of numerical nature, and the second is close to constructive approximation. The first aim is to extend the power of numerical Fourier analysis and to show by means of theoretical examples and many concrete realizations and applications that the usual kernel matrix of the Fourier transform, the discrete Fourier transform (DFT), should be replaced by another kernel matrix, the eXtended Fourier transform (XFT), when computing discrete Fourier transforms of periodic and nonperiodic functions. The XFT kernel appears as a quadrature of a more general transform, the well-known fractional Fourier transform, and furthermore, it can be used as a discretization of the even more general transform, the linear canonical transformation. Our second objective is to present the XFT matrix as a finite-dimensional transformation that links some discrete operators in the same form in which the corresponding continuous operators are related by the Fourier transform, generating sequences of matrix operators representing continuum operators, providing room for studying them from other perspective.

Thus, our aim is to present the theory behind the XFT quadrature and to show the way in which it can be applied to solve problems in many areas, extending in this way the range of applications of discrete Fourier analysis.

The first part of the book is devoted to the standard discrete Fourier transform. The theory of the discrete XFT kernel is presented in Chap. 3. Some applications of this algorithm are given in Chap. 4, including applications to evolution and fractional differentiation problems. The last chapter is devoted to give some new applications of the XFT as a discretization of the fractional Fourier and linear canonical transforms. The contents are based in part on some results that have been published elsewhere, but most of the examples and much of the theory given here are totally new.

Many people have contributed in some way to this book. Among these, Dr. J. Rico-Melgoza and Dr. E. Chavez need a special mention because they are co-developers of the XFT algorithm. Other contributions have come from undergraduate students. I thank all of them for their direct or spontaneous help to solve some of these problems. Some names are Saúl Duarte, Luis Juárez, Lucina Arce, Modesto Pineda, Dionicio Flores, Claudio Meneses, Francisco Mejía, Erick Coronado, Jared Figueroa, Rafael García, Eduardo González, Alejandra Ruiz, Guadalupe Solorio, Marisol López, Paulina Villalón, and Nancy Magaña. Again, thanks to everyone for their help.

Another special mention has to be made: the author gratefully acknowledges Prof. H. G. Feichtinger for the critical reading of the manuscript, his observations, and many advices.

This work was partially supported by the National Southeast Supercomputing Laboratory, project 2018-01-011n, and by the National Council of Science and Technology, project 99006-CB-2008-01.

Chetumal, Mexico
2019

Rafael G. Campos

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