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Linear and Quasilinear Parabolic Problems

Volume II: Function Spaces

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Nil humani quin corrigi possit[1]
L. Naumann
Systematik der Kochkunst
Dresden 1887

Preface

In this volume we present a systematic and detailed exposition of the theory of function spaces in an Euclidean setting. Particular emphasis is put on Besov and Bessel potential spaces which form the frame for the study of parabolic differential equations in the next volume.

The presentation includes several new features which lets it stand out from other accounts. First, it consistently develops anisotropic spaces. Second, it expounds the whole theory for functions and distributions taking their values in Banach spaces on which we impose only the necessary restrictions. Thus none in the case of Besov spaces, except for reflexivity assumptions in duality theorems. Third, the theory is set forth for spaces whose elements are defined on rectangular corners of Euclidean spaces. By this we pave the way for the investigation of function spaces on Riemannian manifolds, possibly possessing corners and other singularities. This is also put on hold for the third volume.

Our approach builds basically on two cornerstones: on Fourier analysis and multiplier theorems, and on extension-restriction techniques. By this we can give a unified presentation incorporating, in particular, Sobolev–Slobodeckii and Hölder space scales. The rather detailed study of these spaces, which are of great importance for the investigation of differential equations, is a further characteristic trait of our treatise.

This volume consists of three chapters and an appendix. The first chapter, which is of rather technical nature, collects preparatory material. It supplies a firm basis for the main text which covers Chapters VII and VIII. The first one thereof contains a systematic treatment of anisotropic vector-valued function spaces on corners. In the second one we give a detailed and unified account of trace and boundary operators.

For the reader's convenience, in the appendix we include a downscaled version of L. Schwartz' theory of vector-valued distributions by admitting only Banach spaces as targets. Particular weight is given on tensor products and convolutions since, in the main text, we make use of such results. It should be mentioned that, already in 2003, I had put a preliminary, slightly more comprehensive version of this appendix on my homepage.

[1]There is nothing on earth that could not be improved.

In essence, this volume forms a profound expansion and amelioration of my earlier lecture notes 'Anisotropic Function Spaces and Maximal Regularity for Parabolic Problems. Part 1: Function Spaces' [Ama09]. Besides of adding much more material, I have corrected numerous flaws and imprecisions which observing readers have brought to my attention.

Once more, I could rely on the help of Pavol Quittner, Gieri Simonett, and Christoph Walker. They read critically and carefully large parts of the first draft, pointed out plenty of mistakes and misprints, and suggested very advantageous changes and improvements. Sincere thanks are given to all of them for their generous support.

Last but not least, I could again experience the immensely valuable support of my wife Gisela who transformed countless barely readable preliminary versions and revisions into TEX files and provided this perfect layout on hand. I am more than deeply grateful to her.

Zürich, January 2019 Herbert Amann

Contents

Notations and Conventions

We use the notations and conventions introduced on pages 1–7 of Volume I, with the following *exceptions*, applied in the main body of this volume (but not in the appendix):

- All abstract vector spaces are over the complex field.
- $\mathbb{K}$ does *not* stand for either the real or the complex number field.
  Instead, it symbolizes corners which are introduced in Subsection VI.1.1.

For the reader's convenience, we reproduce here some of the very basic notation established in Volume I.

Let X and Y be nonempty sets. Then Y^X is the set of all maps $u : X \to Y$. If A is a subset of X, the characteristic function of A (in X) is denoted by χ_A. We also put $\mathbf{1} := \mathbf{1}_X := \chi_X$.

The set of all $x \in X$ in which definitions and relations hold is often denoted by $[\dots]$, where $\dots$ stand for the definitions and relations. For example, if $u \in \mathbb{R}^X$ then

$$[u \geq 0] := \{\, x \in X \ ;\ u(x) \geq 0 \,\}$$

etc.

Suppose X and Y are Hausdorff topological spaces. Then $C(X,Y)$ is the set of all continuous maps in Y^X, endowed with the compact-open topology. We write

$$X \hookrightarrow Y \quad \text{or} \quad i : X \hookrightarrow Y \ ,$$

if X is continuously injected in Y, that is, $X \subset Y$ and the natural injection

$$i : X \to Y \ , \quad x \mapsto x$$

is continuous. If X is a dense subset of Y, we write $X \overset{d}{\subset} Y$. Thus $X \overset{d}{\hookrightarrow} Y$ means that X is densely and continuously injected in Y.

We often write 1_X, or simply 1, for the identity mapping, $\mathrm{id}_X : X \to X$, $x \mapsto x$, if no confusion seems likely.

Given a subset M of a vector space, we put

$$\dot{M} := M \backslash \{0\} \ .$$

Assume X and Y are topological vector spaces. Then $\mathcal{L}(X,Y)$ is the vector space of all continuous linear maps from X in Y, and

$$\mathcal{L}(X) := \mathcal{L}(X,X) \ .$$

In this case $X \hookrightarrow Y$ means also that X is a vector subspace of Y, that is, i belongs to $\mathcal{L}(X,Y)$. Moreover,

$$\mathcal{L}\mathrm{is}(X,Y) := \{\, T \in \mathcal{L}(X,Y) \ ;\ T \text{ is bijective and } T^{-1} \in \mathcal{L}(Y,X) \,\}$$

is the set of all topological linear (toplinear) isomorphisms from X onto Y, and

$$\mathcal{L}\mathrm{aut}(X) := \mathcal{L}\mathrm{is}(X, X)$$

is the group of all toplinear automorphisms of X, the general linear group, also denoted by $\mathcal{GL}(X)$.

By a locally convex space (LCS) we mean a Hausdorff locally convex topological vector space. If X and Y are LCSs, then $\mathcal{L}(X, Y)$ is equipped with the bounded convergence topology.

Throughout this volume, unless specified otherwise, c denotes constants ≥ 1 which may be different from occurrence to occurrence, but are always independent of the free variables appearing at a given place.