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# Advanced Procrustes Analysis Models in Photogrammetric Computer Vision

 Springer

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# Preface

At the beginning of the 1980s, as Assistant Professor of Surveying at the Faculty of Engineering of University of Trieste, I was involved in some research problems relating to the optimal design of geodetic control networks. The optimization model that I introduced at that time foresaw the optimal rotation of the eigenvector matrix of a  $n$ -dimensional variance–covariance matrix of unknown 2D point coordinates of a control network in such a way to dispose the bidimensional point components of the original eigenvector matrix along directions as much as possible orthogonal to those of the forecast deformations. The identification of a computational tool, able to determine the optimal  $n$ -dimensional rotation matrix (with  $n > 3$ ), was not easy. Fortunately, I was guided by University of Trieste statisticians to some books of multifactorial analysis where I could discover a chapter devoted to the so-called *Orthogonal Procrustes Analysis*, a set of algebraic least squares models used to directly perform transformations among corresponding matrix elements, that allowed me to easily solve my optimization problem.

At the end of the 1990s, on the occasion of the 60th birthday celebration of Prof. Erik Grafarend, I was asked to present at the two-day symposium “Quo vadis Geodesia?” a short introduction on Procrustes Analysis in geodetic sciences. This opportunity offered me the possibility to further deepen this topic and to glimpse the important properties of such algorithms for the direct solution of datum transformation problems in geodesy involving the determination of unknown rotations, translations, and isotropic scale factors. Subsequent studies allowed ing. Alberto Beinat and myself to apply some new isotropic Procrustes algorithms for the solution of the photogrammetric block adjustment by independent models, for the conformal updating of a cadastral map, and for the alignment of LiDAR point clouds. These last experiments took advantage also of the valuable contribution of Prof. Domenico Visintini. More recently, the development of innovative algorithms, that consider anisotropic Procrustes models, have permitted to Prof. Andrea Fusiello and myself to find an innovative solution to the image exterior orientation problem and to its extension to the photogrammetric bundle block adjustment. Finally, Dr. Eleonora Maset was capable to find the Procrustes solution of the exterior orientation problem starting from an *Errors-In-Variables* (EIV) model, i.e.,

for a model in which both sets of observations, image and external point coordinates, are characterized by error components.

Although the theme of Procrustes Analysis constitutes the argument of some volume chapters recently written on the topics of advanced algebraic computational problems in geodesy and geoinformatics (Awange and Grafarend 2005; Awange et al. 2010; Awange and Paláncz 2016), in the scientific literature it is not present, at the moment, a volume treating in a complete and thorough manner the analytical developments and the most significant applications of Procrustes Analysis in the field of photogrammetric computer vision and laser scanning.

The aim of this volume is therefore to collect under a unique roof the analytical models developed and experimented by the members of the research group *Geomatics and Computer Vision* of the Polytechnic Department of Engineering and Architecture of University of Udine in photogrammetric computer vision and laser scanning. The first part of the volume is devoted to the theory of Procrustes Analysis, with a detailed description of the different Procrustes models proposed and applied by the authors in the last decades. Before passing to the experimental part, some information about the basics of computer vision and some aspects of laser scanning technology and operational methodology are presented. This allows to introduce classical problems in photogrammetric computer vision and laser scanning that can be effectively solved by various Procrustes models, such as image exterior orientation, the photogrammetric bundle block and independent model adjustments and the LiDAR point cloud registration. Results obtained applying Procrustes Analysis in these fields are presented in the third part of this volume, showing also the robust version of some of the studied algorithms. The illustrated examples allow to highlight the potentiality of the proposed methods for the direct solution of nonlinear systems of equations, without any linearization of the equation systems, nor the knowledge of approximate values of the unknown parameters.

These days, characterized by a rapid growing of sensor system automation and miniaturization and by the availability of huge computational resources, we are witnessing a very fast methodological evolution of problem-solving tools in the field of photogrammetric computer vision and laser scanning. For this reason, the authors of this volume warmly wish the readers to enter into the fascinating field of Procrustes Analysis and to benefit of the many computational advantages offered by this technique.

Finally, as Head of the research group *Geomatics and Computer Vision* of the Polytechnic Department of Engineering and Architecture of University of Udine, I wish to express my gratitude to the group members for the effort put for the publication of the book. In particular, I need to thank Dr. Eleonora Maset, young Ph.D., for leading in a very effective way all the phases of the book assembly.

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# Contents

<b>1</b>	<b>Introduction</b> .....	1
	References .....	3
<b>Part I Theory of Procrustes Analysis Models</b>		
<b>2</b>	<b>Orthogonal Procrustes Analysis</b> .....	7
2.1	The Origins of Procrustes Analysis .....	7
2.2	Classification of the Procrustes Models .....	8
2.3	OPA—Orthogonal Procrustes Analysis .....	9
2.4	EOPA—Extended Orthogonal Procrustes Analysis .....	11
2.5	WEOPA—Weighted Extended Orthogonal Procrustes Analysis .....	14
2.6	OPA and EOPA with Missing Data .....	17
2.6.1	The Case of Missing Data Dimensions .....	17
2.6.2	The Case of Missing Point Coordinates .....	19
2.6.3	The Case of Missing Components .....	20
2.7	Procrustes Statistics .....	21
2.7.1	Procrustes Statistics with Errors in One Configuration . . .	21
2.7.2	Procrustes Statistics with Errors in Two Configurations . . .	22
2.7.3	Analysis of Variance for the Procrustes Statistics .....	23
2.8	Summary .....	25
	References .....	25
<b>3</b>	<b>Generalized Isotropic Procrustes Analysis</b> .....	29
3.1	Basic Hypothesis and Layout of the Problem .....	29
3.2	Original GPA Solution .....	30
3.3	Alternative GPA Solution .....	31
3.4	Weighted GPA with Missing Values .....	34
3.5	Properties of the Centroid .....	37
3.6	Closest Point Generalized Procrustes Analysis .....	39



3.7	Generalized Procrustes Statistics and the Analysis of Variance . . . . .	40
3.8	Summary . . . . .	42
	References . . . . .	43
<b>4</b>	<b>Anisotropic Procrustes Analysis . . . . .</b>	<b>45</b>
4.1	Problem Formulation . . . . .	45
4.2	AEOPA—Anisotropic Extended Orthogonal Procrustes Analysis with Row Scaling . . . . .	46
4.3	AGPA—Anisotropic Generalized Procrustes Analysis . . . . .	49
4.4	Summary . . . . .	50
	References . . . . .	51
<b>5</b>	<b>Procrustes Errors-In-Variables Solutions . . . . .</b>	<b>53</b>
5.1	Errors-In-Variables Model and Total Least Squares Solutions . . . . .	53
5.2	Total Least Squares Solutions of Procrustes Models . . . . .	54
5.2.1	EIV-Extended Orthogonal Procrustes Analysis (EIV-EOPA) . . . . .	55
5.2.2	EIV-Weighted Extended Orthogonal Procrustes Analysis (EIV-WEOPA) . . . . .	58
5.2.3	EIV-Anisotropic Orthogonal Procrustes Analysis (EIV-AEOPA) . . . . .	61
5.3	Summary . . . . .	64
	References . . . . .	64
<b>Part II An Introduction to Computer Vision and Laser Scanning</b>		
<b>6</b>	<b>Basics of Computer Vision . . . . .</b>	<b>67</b>
6.1	Background . . . . .	67
6.1.1	Collinearity Equations . . . . .	70
6.1.2	Camera Resection by DLT . . . . .	71
6.2	Stereo Processing . . . . .	72
6.2.1	Intersection (or Triangulation) . . . . .	72
6.2.2	Relative Orientation and the Essential Matrix . . . . .	74
6.2.3	Two-step Combined Orientation . . . . .	77
6.2.4	Separate Exterior Orientation . . . . .	78
6.2.5	Combined Single Stage Orientation . . . . .	79
6.3	Block Processing . . . . .	80
6.3.1	Bundle Block Adjustment . . . . .	81
6.3.2	Resection–Intersection Method . . . . .	82
6.3.3	Independent Models Block Adjustment . . . . .	83
6.4	Summary . . . . .	84
	References . . . . .	84

**7 Basics of Terrestrial Laser Scanning** . . . . . 87

7.1 Background . . . . . 87

7.2 Registration and Geo-referencing of Laser Scanning  
Point Clouds . . . . . 90

7.3 Laser Scanning with Moving Systems . . . . . 94

7.4 Summary . . . . . 96

References . . . . . 96

**Part III Applications of Procrustes Analysis Models**

**8 Applications of Anisotropic Procrustes Analysis** . . . . . 101

8.1 Solving the Exterior Orientation Problem with AEOPA . . . . . 102

8.1.1 Background and Related Work . . . . . 102

8.1.2 Exterior Orientation with Fiore’s Algorithm . . . . . 103

8.1.3 Exterior Orientation with AEOPA . . . . . 104

8.1.4 Experimental Validation . . . . . 107

8.1.5 EIV-AEOPA Solution for the Exterior Orientation  
Problem . . . . . 110

8.1.6 Robust Procrustes Exterior Orientation . . . . . 112

8.2 Solving the Non-Perspective-n-Point Problem with AEOPA . . . . . 118

8.2.1 NPnP Formulation . . . . . 120

8.2.2 Procrustes Solution of the NPnP Problem . . . . . 121

8.2.3 Algebraic, Direct Solution . . . . . 123

8.2.4 Experimental Validation . . . . . 124

8.3 Solving the Bundle Block Adjustment Problem with AGPA . . . . . 125

8.3.1 Bundle Block Adjustment with AGPA . . . . . 128

8.3.2 Experimental Validation . . . . . 131

8.3.3 Robust Procrustes Bundle Adjustment . . . . . 139

8.4 Summary . . . . . 147

References . . . . . 147

**9 3D Model Registration by Generalized Procrustes Analysis** . . . . . 153

9.1 Solving the Independent Models Block Adjustment Problem  
with GPA . . . . . 154

9.1.1 Experimental Validation . . . . . 157

9.2 Solving the Point Cloud Registration Problem with GPA . . . . . 161

9.2.1 Global Registration with Known Correspondences . . . . . 164

9.2.2 Embedding the Correspondence Estimation  
Inside GPA . . . . . 164

9.3 Summary . . . . . 172

References . . . . . 172