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# Energy Transfers in Atmosphere and Ocean

Springer

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ISSN 2524-4264 ISSN 2524-4272 (electronic)
Mathematics of Planet Earth
ISBN 978-3-030-05703-9 ISBN 978-3-030-05704-6 (eBook)
https://doi.org/10.1007/978-3-030-05704-6

Library of Congress Control Number: 2018964245

Mathematics Subject Classification (2010): 35Q35, 35Q75, 35Q86, 65Z05, 86-08, 86A05, 86A10, 86A17

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# Preface

This book describes a recent effort combining interdisciplinary expertise within the Collaborative Research Centre

“Energy Transfers in Atmosphere and Ocean” (TRR 181),

which was funded by the German Research Foundation (DFG).

Energy transfers between the three dynamical regimes—small-scale turbulence, internal gravity waves and geostrophically balanced motion—are fundamental to the energy cycle of both the atmosphere and the ocean. Nonetheless, they remain poorly understood and quantified and have yet to be adequately represented in today’s climate models. Since interactions between the dynamical regimes ultimately link the smallest scales to the largest ones through a range of complex processes, understanding these interactions is essential to constructing atmosphere and ocean models and to predicting the future climate.

The current lack of understanding is reflected by energetically inconsistent models with relatively large biases, but also by inconsistencies of a numerical and mathematical nature. In TRR 181, recent efforts to overcome these deficiencies are combined, and new endeavours to understand dynamical interactions and to improve the consistency of ocean and atmosphere models are fostered.

The point of departure for TRR 181 is the recognition that the energy cycle is inconsistently represented in current climate models, i.e. that the models are energetically inconsistent. A primary example of this inconsistency is the effect of the dissipation of the unresolved internal gravity wave field in the ocean, which is parameterised in standard models by mixing of density with a prescribed diffusivity. Although this diffusivity is sometimes linked to resolved parameters or to the energy input into the internal wave field, a consistent description of the energetics of the internal wave field is usually lacking. The same is true for gravity wave drag parameterisation in atmosphere models and, more generally, for nearly all parameterisations which are used today: it applies to the parameterisation of dissipation of the (available) potential energy of the turbulent balanced flow in ocean models by (isopycnal) layer thickness diffusion, the dissipation of resolved kinetic energy by hyper-viscosity in atmosphere and ocean models, and the dissipation of energy in bottom boundary layers in both types of model. For all of these processes, the energy that is dissipated is simply lost instead of being transferred to the relevant connecting dynamical regime, or to a different form of energy; this represents a previously overlooked flaw for which solutions are only now beginning to emerge, as described in this book.

That being said, in other contexts and for other parameterisations, this "missing" energy must be artificially recreated. A prominent example is the unaccounted-for (wave-driven) supply of energy needed to mix density in the ocean by prescribing some interior diffusivity. Another example is the heating of the upper atmosphere by molecular dissipation—also related to gravity wave breaking—but the same holds true for virtually any other parameterisations and dynamical regime. In other words, our current atmosphere and ocean models fail to completely account for the mechanical energy cycle. The goal of TRR 181 is to remedy this shortcoming by connecting all parameterisations in state-of-the-art atmosphere and ocean models in an energetically and mathematically consistent way. To this end, 16 sub-projects with 29 principal investigators from applied mathematics, meteorology, and physical oceanography have been established. The following chapters provide an overview of representative specific topics covered by the sub-projects.

The processes, parameterisations, and interactions addressed range from isolated idealised model setups to fully coupled global climate models. Chapter 1 starts out by reviewing a coherent hierarchy of models for the dynamical core in which many of the multi-scale interactions (which also repeatedly feature in the later chapters of this volume) are explored. Due care is taken to fully specifying the respective scaling and simplifying assumptions and to exposing the underlying Hamiltonian structure of the inviscid equations. The chapter subsequently discusses modifications to the classical scaling regimes in the equatorial region, and the impact of different forms of viscous dissipation or eddy damping on the solution and bifurcation structure of geophysical flows. Lastly, the chapter covers the systematic derivation of stochastic parameterisations for small-scale motions, a major theoretical development of the last decade which is yet to be fully understood and to be investigated operationally.

A major point of conceptual and practical uncertainty in the modelling of the atmosphere and the ocean is the dissipation of large-scale mean or eddying balanced flow, for which several processes have been put forward. They involve interior loss of balance by either ageostrophic or symmetric instability, Lighthill radiation of gravity waves, lee wave generation at topographic obstacles, interaction with western boundaries, the interaction between gravity waves and frontogenesis, or simply bottom friction. The relative importance of these processes for both the atmosphere and the ocean, however, has yet to be quantified, despite the fact that the dissipation of balanced flow is a key element in the energy cycle. Accordingly, Chapter 2 is devoted to this topic.

Momentum exchange between the different dynamical regimes, for instance the wave drag forcing of large-scale circulation in the upper atmosphere by upward propagating and breaking gravity waves, is also important to understanding the mechanism of circulation and the energy cycle. However, wave breaking, the interaction of waves with the mean flow and stratification, and especially wave–wave interactions (and the resulting spectral energy transfers within the wave field to smaller wavelengths and thus towards wave breaking and density mixing) are only poorly understood, but essential to grasping the effects of gravity waves on the large-scale circulation. Chapters 3 and 4 focus on the topic of gravity waves, in the ocean and atmosphere, and in models and observations.

While the turbulent energy cascade in the classical isotropic turbulence regime at high Reynolds numbers appears to offer a valid description, the assumption of an energy transport between neighbouring wave numbers in an inertial sub-range over several decades without any dissipation effects appears invalid for turbulent flow on larger scales, in particular for geostrophic turbulence. As such, basing parameterisations on this assumption is problematic. Spectral energy transport from small to large scales is present, but usually ignored in standard sub-grid closures such as harmonic, hyper- or non-linear viscosity. In particular, the use of harmonic lateral friction in ocean models for purely numerical reasons is in striking contradiction to an inverse energy cascade and underscores the need to reconsider present sub-grid-scale closures in ocean and atmosphere models. New approaches to solving these problems are described in Chapter 5.

Beyond energy, there are also other properties of the system that require consistent treatment. Amongst them is momentum conservation, which puts constraints on, e.g. eddy parameterisation in ocean models, since eddies are known to redistribute, but not to create momentum. Eddy parameterisations for the ocean and the challenges in the diagnosis of eddy effects from models and observations are discussed in Chapter 6. A further issue of fundamental physical importance is addressed in Chapter 7, which focuses on the second law of thermodynamics and how it can be consistently related to the averaged equations of motion and in particular their sub-grid closures and on how parameterisations such as hyper-viscosity can be made consistent using what we know about the directions of energy cascades from large to small scales.

The physical inconsistencies in current state-of-the-art modelling are paralleled by numerical inconsistencies and challenges. While advection conserves all properties of a fluid particle, numerical advection schemes are by definition notoriously non-conservative, since they introduce spurious mixing and dissipation, a well-known but serious shortcoming of current atmosphere and ocean models. This issue is at the heart of Chapter 8. On the one hand, this spurious mixing and dissipation needs to be reduced as much as possible using advanced numerical techniques, such as mesh adaptivity or mesh-free Lagrangian methods. On the other, the remaining inescapably spurious energy sources also need to be quantified and should be taken into account when designing new parameterisations and sub-grid closures. In turn, air–sea interaction is another fundamental but numerically challenging aspect of the climate system. Surface ocean waves play a key role in transferring momentum, energy, heat, and other properties across the air–sea boundary; yet, their interaction with large- and small-scale flows in the ocean and atmosphere is not well understood. Chapter 9 details a promising new approach to directly simulating these processes numerically.

In TRR 181, the process-oriented topics presented here are complemented by an operationally oriented synthesis focusing on two climate models currently being developed in Germany. In this way, the goal of TRR 181 is to help reduce the biases in and increase the accuracy of atmosphere and ocean models and ultimately to improve climate models and climate predictions.

Hamburg, Germany
October 2018

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# Acknowledgements

The chapters of this work refer to projects pursued at the Collaborative Research Centre “Energy Transfers in Atmosphere and Ocean” (TRR 181), funded by the German Research Foundation (DFG). Moreover, the encouraging support of Martin Peters (Springer Heidelberg), which was the initial spark for this book, is gratefully acknowledged.

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