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Davide Crippa

The Impossibility of Squaring the Circle in the 17th Century

A Debate Among Gregory,
Huygens and Leibniz

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Preface

This book is the first outcome of a research programme in the study of impossibility results in mathematics. I began this programme in my doctoral dissertation, of which this book is an abridged and improved version. I am indebted to the many people whose assistance allowed this book to come into existence: first and foremost, Marco Panza, whose helpful and careful guidance made this work possible, and Vincenzo de Risi, who gave me the concrete opportunity to publish it. I also heartily thank Andy Arana, Andrey Bovykin, Abel Lassalle Casanave, Jesper Lützen, Bodo von Pape, Jean-Jacques Szczeciniarz, and David Rabouin for their invaluable suggestions, help and criticism.

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Paris, France
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Davide Crippa

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