Discrete Calculus

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Applied Analysis on Graphs for Computational Science

Springer

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Leo (John) Grady dedicates the book to his very patient wife, Amy Grady, daughter Alexandra Grady, son Leo James Grady (who arrived during the course of writing) and the memory of his late father, Leo Joseph Grady.

Preface

The goal of this book is to present the topic of discrete calculus to scientists and engineers and to show how the theory can be applied to solving a wide variety of real-world problems. We feel that discrete calculus allows us to unify many approaches to data analysis and content extraction while being accessible enough to be widely applied in many fields and disciplines. This project initially began as a tutorial on discrete calculus and its applications, and we hope that this work can provide an introduction to discrete calculus and demonstrate its effectiveness in many problem domains.

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Princeton, New Jersey — Leo Grady
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Contents

Acronyms

AC	Alternating Current
BEM	Basic Energy Model
CDP	Constitutive Determination Problem
CFL	Courant–Friedrichs–Lewy
CRT	Chromatographic Retention Time
CT	Computed Tomography
DC	Direct Current
DFT	Discrete Fourier Transform
DNA	Deoxyribonucleic Acid
fMRI	functional Magnetic Resonance Imaging
GMRF	Gaussian Markov Random Field
GPU	Graphics Processing Unit
HITS	Hyperlink-Induced Topic Search
IRLS	Iteratively Reweighted Least Squares
KCL	Kirchhoff's Current Law
KM	K-Means
KVL	Kirchhoff's Voltage Law
LE	Laplacian Eigenmaps
LII	Legal Information Institute
LP	Linear Programming
LPP	Locality Preserving Projections
LTI	Linear Time Invariant
MAP	Maximum *A Posteriori*
MDS	Multidimensional Scaling
ML	Maximum Likelihood
MRF	Markov Random Field
MRI	Magnetic Resonance Imaging
MPCV	MultiPhase Chan–Vese
mRNA	messenger Ribonucleic Acid
MS	Mumford–Shah
MSF	Maximum Spanning Forest

NP-Hard	Non-deterministic Polynomial-Time Hard
PCA	Principle Components Analysis
PDE	Partial Differential Equation
QP	Quadratic Programming
QSAR	Quantitative Structure–Activity Relationships
QSPR	Quantitative Structure–Property Relationships
RGB	Red Green Blue
RLC	Resistor–Inductor–Capacitor
SDP	Semidefinite Programming
SNF	Smith Normal Form
SPD	Symmetric Positive Definite
SPECT	Single Photon Emission Computed Tomography
TV	Total Variation