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John Vince

Mathematics for Computer Graphics

3rd Edition

Springer

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This book is dedicated to my grandchildren:
Megan, Mia and Lucie.

Preface

Mathematics is a beautiful subject. Its symbols, notation and abstract structures permit us to define, manipulate and resolve extremely complex problems. However, the symbols by themselves are meaningless – they are nothing more than a calligraphic representation of a mental idea. If one does not understand such symbols, then the encoded idea remains a secret.

Having spent most of my life using mathematics, I am still conscious of the fact that I do not understand much of the notation used by mathematicians. And even when I feel that I understand a type of notation, I still ask myself "Do I really understand its meaning?" For instance, I originally studied to be an electrical engineer and was very familiar with $\mathrm{i} = \sqrt{-1}$, especially when used to represent out-of-phase voltages and currents. I can manipulate complex numbers with some confidence, but I must admit that I do not understand the physical meaning of i^{i}. This hole in my knowledge makes me feel uncomfortable, but I suppose it is reassuring to learn that some of our greatest mathematicians have had problems understanding some of their own inventions.

Some people working in computer graphics have had a rigorous grounding in mathematics and can exploit its power to solve their problems. However, in my experience, the majority of people have had to pick up their mathematical skills on an *ad hoc* basis depending on the problem at hand. They probably had no intention of being mathematicians, nevertheless they still had to study mathematics and apply it intelligently, which is where this book comes in.

To begin with, this book is not for mathematicians. They would probably raise their hands in horror about the level of mathematical rigour I have employed, or probably not employed! This book is for people working in computer graphics who know that they have to use mathematics in their day-to-day work, and don't want to get too embroiled in axioms, truths and Platonic realities.

This book originally appeared as part of Springer's excellent *"Essential"* series, and was revised to include chapters on analytical geometry, barycentric coordinates and worked examples. This edition includes a new chapter on geometric algebra, which I have written about in my books *Geometric Algebra for Computer Graphics* and *Geometric Algebra: An Algebraic System for Computer Games and Animation*.

Although I prepared the first book using Microsoft WORD, for this last edition I have used LaTeX 2ε which has greatly improved the layout. This, however, has required me to type in every equation again, which was not only tedious, but an opportunity to correct a handful of typos that always seem to find there way into books. I have also redrawn all the illustrations to bring a consistent graphical appearance to the book. LaTeX 2ε is an amazing software system – extremely fast and robust. The entire book only takes 4 s to typeset, which permitted me to edit the final draft and recompile every time I changed a single punctuation mark!

Whilst writing this book I have borne in mind what it was like for me when I was studying different areas of mathematics for the first time. In spite of reading and rereading an explanation several times it could take days before "the penny dropped" and a concept became apparent. Hopefully, the reader will find the following explanations useful in developing their understanding of these specific areas of mathematics, and enjoy the sound of various pennies dropping!

Once again, I am indebted to Beverley Ford, General Manager, Springer UK, and Helen Desmond, Assistant Editor for Computer Science, for persuading me to give up holidays and hobbies in order to complete this book! I would also like to thank Springer's technical support team for their help with LaTeX 2ε.

Ringwood, John Vince
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