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Optimal Linear Controller Design for Periodic Inputs

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Preface

Periodic reference and disturbance signals are widespread in engineering practice, as every rotating machine and repeated process involves periodicity. Exploiting the periodic input characteristics in the controller design is indispensable to meet tight performance demands in spite of measurement noise, model inaccuracies…

This monograph proposes a general design methodology for linear controllers facing periodic inputs, which applies to all controller types reported in the literature. The proposed design methodology is able to reproduce and outperform major current design approaches, where this superior performance stems from the following properties: (i) uncertainty on the input period is explicitly accounted for; (ii) periodic performance is traded-off against conflicting design objectives; and (iii) the controller design is translated into a convex optimization problem, guaranteeing the efficient computation of its global optimum. Apart from extensive numerical evaluation, the potential of the design methodology is experimentally illustrated on an active air bearing setup.

This monograph is the result of four years of PhD research at the Division of Production Engineering, Machine Design & Automation (PMA), Department of Mechanical Engineering, Katholieke Universiteit Leuven, Belgium. I am grateful to all people who contributed to this work. I would like to give a special word of thanks to my supervisors Jan Swevers and Bram Demeulenaere for offering me support and guidance, while at the same time giving me the freedom to choose my research niche. I wish to acknowledge Z. Liu and Prof. L. Vandenberghe (UCLA, Electrical Engineering Department) for their kind assistance and pertinent comments concerning the numerical solution of the SDPs involved. Also many thanks to the Research Foundation–Flanders (FWO–Vlaanderen) for providing a fellowship for this research. Last but not least, I am indebted to my parents who always encouraged and supported me.

Leuven, Belgium,
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Goele Pipeleers

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Abbreviations

Sets

$\mathbf{R}$	real numbers
$\mathbf{R}_n$	real n-vectors
$\mathbf{R}_{n\times m}$	real $n \times m$ matrices
$\mathbf{C}$	complex numbers
$\mathbf{C}_n$	complex n-vectors
$\mathbf{C}_{n\times m}$	complex $n \times m$ matrices
$\mathbf{S}_n$	symmetric $n \times n$ matrices
$\mathbf{H}_n$	Hermitian $n \times n$ matrices

General Symbols

I_n	$n \times n$ identity matrix, where the subscript is omitted if the dimension is clear from the context
$0_{n\times m}$	$n \times m$ zero matrix, where the subscript is omitted if the dimension is clear from the context
X^T	transpose of matrix X
X^H	Hermitian (complex conjugate) transpose of matrix X
$\mathrm{Tr}\{X\}$	trace of matrix X
$\otimes$	matrix Kronecker product
$\|X\|$	cardinal number of finite set X
$\sigma_{\max}\{X\}$	largest singular value of matrix X
$\Re\{X\}$	real part of X
$\Im\{X\}$	imaginary part of X

Discrete-time, Linear Time-invariant Systems

f_s	sample frequency [Hz]
T_s	sample period [s]: $T_\mathrm{s} = 1/f_\mathrm{s}$
q	one-sample-advance operator
z	discrete-time Laplace variable
k	index labeling the sampled time instants kT_s
ω	frequency [rad/s]
$X(q)$	difference equation of system X
$X(z)$	transfer function (matrix) of system X
$X(\omega)$	FRF (matrix) of system X
$X_+(z)$	noninvertible part of SISO transfer function $X(z)$, comprising a delay equal to the relative degree of $X(z)$ and its nonminimum-phase zeros
$X_-(z)$	invertible part of SISO transfer function $X(z)$: $X(z) = X_+(z)X_-(z)$
$\|X(z)\|_\infty$	$\mathscr{H}_\infty$ norm of system X

Control Configuration

$w(k)$	exogenous input
$u(k)$	control input
$v(k)$	regulated output
$y(k)$	measured output
$r(k)$	reference input
$d(k)$	disturbance input
$\eta(k)$	plant output
$e(k)$	tracking error: $e(k) = r(k) - \eta(k)$
G	plant
P	generalized plant
K	controller
H	closed-loop system
S	closed-loop sensitivity
T	closed-loop complementary sensitivity

Periodic Input

T_p	nominal value of the period [s]
f_p	nominal value of the fundamental frequency [Hz]: $f_\mathrm{p} = 1/T_\mathrm{p}$
ω_p	nominal value of the fundamental frequency [rad/s]: $\omega_\mathrm{p} = 2\pi f_\mathrm{p}$
$\boldsymbol{\delta}$	relative uncertainty on f_p and ω_p
$f_{\mathrm{p},\delta}$	potential values of the fundamental frequency [Hz]: $f_{\mathrm{p},\delta} = f_\mathrm{p}(1+\delta)$, where $\|\delta\| \leq \boldsymbol{\delta}$
$\omega_{\mathrm{p},\delta}$	potential values of the fundamental frequency [rad/s]: $\omega_{\mathrm{p},\delta} = 2\pi f_{\mathrm{p},\delta}$
l	index labeling the harmonics
$\mathscr{L}$	set of harmonics to be suppressed

$n_{\mathscr{L}}$	number of elements in $\mathscr{L}$
W_l	positive weight quantifying the relative importance of the l'th harmonic in the periodic input
Ω_l	uncertainty interval on the l'th harmonic frequency [rad/s]: $\Omega_l = \left[l\omega_{\mathrm{p}}(1-\boldsymbol{\delta}) \, , \, l\omega_{\mathrm{p}}(1+\boldsymbol{\delta}) \right]$
Λ	signal generator of the periodic input, for nominal period T_{p}
n_{Λ}	order of Λ

Abbreviations

FIR	finite impulse response
FRF	frequency response function
KYP	Kalman-Yakubovich-Popov
LMI	linear matrix inequality
LTI	linear time-invariant
MIMO	multiple-input multiple-output
rms	root-mean-square
SDP	semi-definite programming problem
SISO	single-input single-output
SOCP	second-order cone problem