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Introduction to Analytical Dynamics

New Edition

Springer

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Preface to the New Edition

This is a revised edition of a text on classical mechanics that was originally published 20 years ago by Oxford University Press. I have taken the opportunity to simplify some of the presentation, while keeping to the original intention of confronting rather than evading the various notational and pedagogical difficulties that one encounters in the journey from Newton to Lagrange and Hamilton. I have also responded to comments over the years from colleagues and, more recently, from the new publisher's referees.

There are two major changes. I have gathered together the material and examples on systems with one degree of freedom into a separate chapter. The intention here is to give a first introduction to the core ideas of the Lagrangian theory in a context in which they make strong contact with familiar elementary techniques from the treatment of ordinary differential equations, without the distraction of indices and the summation convention. Second, I have added a chapter on differential geometric methods.

I am grateful to the many students and colleagues who have commented on the first edition, and pointed out mistakes.

N.M.J. Woodhouse
Oxford, February 2009

Preface to the First Edition

It may seem odd that Newtonian mechanics should still hold a central place in the university mathematics curriculum. But there are good reasons.

– It is one of the most accurate physical theories ever devised. Three hundred years after the publication of Newton's *Philisophiae naturalis principia mathematica* (1687), we should be surprised not that some of his ideas have been superseded by relativity and quantum theory, but that it is still necessary to exercise great subtlety and scientific ingenuity to detect any error at all in the three laws of motion. Even in the prediction of the orbit of the planet Mercury, for example, which was a crucial failure of the classical theory, the discrepancy[1] is only one part in 10^7.

  Newton's theory is the prime example of what Wigner calls the 'unreasonable effectiveness of mathematics' as a tool for understanding the physical world – an aspect of the truth of mathematics that can easily be lost in a course overburdened with abstraction [14].

– Quantum theory and relativity have overthrown the classical view of physics, but the mathematical formalism of classical mechanics still plays an essential part. It provides both a framework for interpretation and a first introduction to key ideas and techniques (frames of reference, general coordinate transformations, the connection between observables and symmetries,...). It is an essential prerequisite for any advanced course on applications of mathematics in modern theoretical physics.

– It develops geometric intuition and gives invaluable practice in problem

[1] The radius vector from the Sun to Mercury sweeps out a total angle of $150,000^\circ$ per century. The prediction of the Newtonian theory is $43''$ less than the observed angle. For the other planets, it is much less.

solving and mathematical modelling. It is easy to poke fun at the seemingly endless supply of light rods, inextensible strings, and smooth hemispheres. But all undergraduate exercises are necessarily artificial, however cleverly they are dressed. The strength of mechanics is the vast range of its examples, something that their familiarity can make us overlook, and the diversity of different mathematical ideas that they illustrate.

– The problems of classical mechanics and, in particular, the centuries of work on planetary motion, stimulated the development of much of modern mathematics. It is no coincidence that the great names of mechanics, Newton, Euler, Poisson, Lagrange, Hamilton,. . . , also occur over and over again throughout many branches of pure mathematics. It is essential to study classical mechanics to understand the roots of mathematics.

– The influence of classical mechanics is still present in modern pure mathematical research. The study of Hamilton's equations, for example, led to the development of symplectic geometry, which in turn has found recent applications in the analysis of partial differential equations and in the representation theory of Lie groups.

A glance through the pages that follow will not reveal anything strikingly unfamiliar. The range of topics is central and traditional, partly because I want the book to be short and (OUP willing) cheap, and partly because I intend it to be no more than an introduction. I hope that it will be read in conjunction with the classics and that it will encourage further exploration (in, for example, Arnol'd's *Mathematical methods of classical mechanics* [1]).

The book is written for second year mathematics undergraduates and assumes familiarity with elementary linear algebra, the chain rule for partial derivatives, and vector mechanics in three dimensions (the last is not absolutely essential). The main intention is, first, to give a confident understanding of the chain of argument that leads from Newton's laws through Lagrange's equations and Hamilton's principle to Hamilton's equations and canonical transformations; and, second, to give practice in problem solving. Most of the exercises and examples are taken from recent Oxford examination papers.

I have concentrated on trying to clarify the points that come up most frequently in tutorials and that I myself found confusing when I first met these ideas. For example: why are you allowed to say that q and $\dot{q}$ are independent? and: why can I not deduce from $\partial L/\partial t = -\partial h/\partial t$ that $h + L$ is independent of t?

It is true, of course, that the most satisfactory way to come to terms with the mathematics of classical mechanics would be to approach the subject from modern differential geometry. But that would mean reducing analytical mechanics to a minority option at the end of the undergraduate course or in the first year of graduate work, which would be a great loss. Instead, I have tried to make use of lessons that I have learnt from differential geometry, but without ever going outside the framework of local, coordinate-based arguments.

I am particularly grateful to Paul Tod and Tom Cooper for many comments on an earlier version of this book; and to Rob Baston, Andy Clark, Mike Dobson, Steve Lloyd, Diana Mountain, Charles Sanderson, and Steve Thorsett for working through the final version.

Oxford 1986
N.M.J. Woodhouse

Note. Examples and exercises marked with a dagger (†) are adapted from examination questions set at the University of Oxford.

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