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Anna Maria Perdon

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# **Algebraic Methods for Nonlinear Control Systems**

**2nd Edition**

 Springer

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To Federica Paola

To Renan-Abhinav

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## Preface to the Second Edition

The first edition of this book appeared at the end of the last century and, since then, it has been used as a textbook in several graduate courses, summer schools, and a preconference tutorial workshop on Nonlinear Systems. Thanks to the experience gained in these activities, several modifications appeared to be appropriate. The chapter on modeling was enlarged and it now includes results on standard realization of nonlinear systems. This is motivated by the importance of state-space representations in the analysis and synthesis of nonlinear control systems in the current literature. The focus of the book remained about structural properties that do not involve stability issues and, for this reason, those issues are only marginally considered. The chapter on systems structure has been enlarged by adding more material on system inversion and by adding motivational examples. A new chapter on output feedback has been included at the end of the book, in view of the major importance it has in practical applications. Various supporting practical examples borrowed from robotics, mechanics, and other application areas have been added throughout the book. A few exercises complete the chapters.

The introduction to the differential algebraic approach has been deleted because new monographs on this topic are available.

Finally, solutions to the problems can be found on the following website: <http://www.springer.com/1-84628-594-1>

Ancona, May 2006,  
Nantes, May 2006,

*Giuseppe Conte and Annamaria Perdon*  
*Claude H. Moog*

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## Preface to the First Edition

The theory of nonlinear control systems owes a large part of its modern development and success to the systematic use of differential geometric methods and tools. One of the first problems to be considered from the point of view of differential geometry was, at the beginning of the 1970s, that of analyzing the controllability of a nonlinear system. Early works on that topic ([111, 154, 155, 9]) highlighted the power and the potentiality of the differential geometric approach and motivated the interest of many researchers.

During the 1980s, the possibilities offered by the use of differential geometric techniques in the study of nonlinear control systems were largely exploited. One of the underlying leading ideas (see [75, 87]) was that of generalizing, to the greatest possible extent, the so-called geometric approach which had been first developed in the linear case (see [6, 160]). The research effort produced, in that period, many important results and it provided effective solutions to several control problems, such as disturbance decoupling problems, non-interacting control problems, and model matching problems. Excellent and comprehensive descriptions of the methodology and of the results achieved, together with meaningful examples of applications, can be found in [86] and in [126].

In the second half of the 1980s the limits of the differential geometric approach started to be explored and to become known. In particular, it became clear that problems such as system inversion or the synthesis of dynamic feedbacks could hardly be tackled with the already well-established differential geometric methods.

In the same period, the introduction of differential algebraic methods in the study of nonlinear control systems ([49]) offered a way to circumvent a number of difficulties encountered up to that time. The use of differential algebraic concepts characterized, through the work of several authors, a novel approach, which has essentially an algebraic nature, and, at the same time, it provided additional tools for investigating old and new problems. Further results on problems pertaining to inversion, noninteracting control, realization

and reduction to canonical forms were obtained in the following years, and others were made achievable.

Today, the use of an algebraic point of view in nonlinear control problems has gained popularity and diffusion. This motivates the present book, whose aim is to give an account of the algebraic approach to nonlinear system theory and of its development in recent years. Together with a number of results which are scattered in the literature, the reader will find in it a self contained, comprehensive description of techniques and tools that can enrich his equipment as a control theorist and can provide a solution to otherwise not easily tractable control problems.

One of the distinctive characteristics that makes the algebraic approach interesting and useful is its inherent simplicity. In comparison with the mathematical background needed for profitably employing differential geometric methods, the knowledge required for using the tools described in this book is very limited. A significant example of this is offered by the way in which the notion of accessibility and the problem of linearization are dealt with. In both cases, a single tool, based on elementary differentiation of a function, namely, the notion of relative degree, gives the key for carrying on a deep analysis and for characterizing relevant dynamic properties. From a didactic point of view, simplicity renders the algebraic approach a practicable and valid choice in teaching engineering courses on nonlinear control. The book emphasizes this aspect and is usable as a teaching aid. In addition, simplicity facilitates the development of efficient algorithmic procedures that are relevant in solving concrete analysis and synthesis problems.

Another positive quality of the algebraic approach is its wide applicability in the field of dynamic systems and control. Although only continuous-time systems are considered in the book, the tools and methods described apply successfully to a number of control problems concerning discrete-time nonlinear systems, as shown in [4, 68]. Applications to time-varying systems are also possible, and recent results have been obtained in dealing with time-delay systems (see [13, 120]). With respect to other general methodologies, then, the algebraic approach appears to be more versatile and capable of going to the heart of the problem.

Only a basic knowledge of systems and control theory is required for reading the book, whose material is arranged in a self-explanatory way. The general setting and the fundamental notions are described and illustrated in the first part, entitled Methodology. Mathematical preliminaries are presented including notations from exterior differentiation. The system analysis completes this part and deals with fundamental properties as accessibility and observability. The structure algorithm and a canonical decomposition of the system are given as well.

In the second part, entitled Applications to Control Problems, the tools and techniques of the algebraic approach are employed for solving a number of basic control problems that are of practical interest in fields such as robotics and control of general mechanical systems, as well as in process control. The



solution of the feedback linearization problem is given in terms of the accessibility filtration  $\{\mathcal{H}_k\}$  introduced in Chapter 3. The disturbance decoupling problem is solved using the subspace  $\mathcal{X} \cap \mathcal{Y}$ , namely, the subspace that is observable independently from the input (Chapter 4). In the noninteracting control problem and in the model matching problem, we use the output filtration  $\{\mathcal{E}_k\}$  and the structure algorithm (Chapter 5).

Finally, in the third part, entitled Differential Algebra, differentially algebraic tools and concepts are introduced in an elementary, but comprehensive, way, and the results of the first parts are revisited and analyzed from a differentially algebraic point of view. The notions described in the third part may contribute to expand not only the technical knowledge of the reader, but also his comprehension of the key ideas of the algebraic approach. A conceptually powerful way of approaching the theory of nonlinear systems has been proposed at the end of the 1980s in [49, 52, 54], introducing the use of differential algebra and differential-algebraic methods. In comparison with other approaches which employ differential geometric methods (see [86, 126] for a comprehensive description) or Volterra or generating series (see [48]), this one appears in particular capable of removing some drawbacks present in the notions of rank and it allows us to characterize general invertible compensators. In addition, it provides useful insight and results in connection with various analysis and synthesis problems (see [44, 52, 54, 64, 136]). Although differential algebraic methods are better suited for dealing with systems described by polynomials or rational functions, extensions to more general cases (as suggested, for instance, in [50, 141]) are possible. Here, we give a brief introduction to the principal notions of differential algebra and we introduce a general notion of dynamic systems in differential algebraic terms. Related system theoretical properties are described and the principal results obtained by the differential algebraic approach are mentioned without entering into the details to help the reader in establishing a connection with the notions studied in the previous parts of this book.

The authors acknowledge the NATO for its financial support of a joint research project under grant CRG 890101. Several results of this project are reported in this book.

Finally, the authors would like to thank J.W. Grizzle, and M.D. di Benedetto for some joint research work which inspired this book. Valuable discussions with M. Fliess, A. Isidori, A. Glumineau, E. Aranda, J.B. Pomet, R. Andiarri, Ü. Kotta, and Y.F. Zheng are acknowledged as well as the careful reading by L.A. Márquez Martínez and R. Pothin, and the help of E. Le Carpentier.

Ancona, December 1998,  
Nantes, December 1998,

*Giuseppe Conte and Annamaria Perdon*  
*Claude H. Moog*

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