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Krzysztof Gałkowski

State-space Realisations of Linear 2-D Systems with Extensions to the General nD (n>2) Case

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Author

Krzyztof Gałkowski, PhD, Habil., Prof

Institute of Control and Computational Engineering, Technical University of Zielona Góra, Podgórna Str. 50, 65-246, Zielona Góra, Poland

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Preface

The field of multidimensional (nD) systems continues to grow and mature. In particular, there are many new approaches and applications areas which are rapidly emerging into the research literature. As example, the behavioural approach to basic systems theoretic questions for nD systems has already proved to a very powerful approach for solving long standing open questions in areas such as controllability and physically relevant definitions of poles and zeros and their implications for, say, the specification and (eventual) design of control schemes.

In the applications domain paper making processes and systems with repetitive dynamics have emerged as areas where adopting an nD systems approach has clear advantages over alternatives.

In the nD systems area, the problem of constructing state-space realizations is of fundamental importance (as it is, of course, for standard (or 1D) systems). The nD case is much harder in relative terms due to complications in the theory of the underlying (polynomial/polynomial matrix based) algebraic structure, such as the distinction in the nD case between factor primeness, minor primeness, and zero primeness, or the lack of a Euclidean algorithm. These problems are particularly acute when seeking the minimal realization.

This monograph gives a comprehensive treatment of the so-called Elementary Operations Approach (EOA) for the construction of a range of state-space realizations for 2D, and more generally, nD linear systems.

In Chapter 1, we give a brief overview of the main features of nD systems, placing particular emphasis on the differences with standard, or 1D, linear systems and currently open research questions. This is supported by references to the relevant literature and an outline of the main aims and contents of this monograph.

Chapter 2 gives a summary of the relevant mathematical tools and algorithms used.

Chapter 3 solves the basic problem of: given a bivariate polynomial, construct its companion matrix. This problem is also used to illustrate the basic operation of the EOA approach.

Chapter 4 then extends these results to the case of bivariate 2D transfer functions, i.e. for single-input single-output (SISO) linear systems.

In Chapter 5, the method is extended to the multiple-input multiple-output (MIMO) linear 1D systems, which is a necessary and non-trivial step before the 2D MIMO case in Chapter 6.

In Chapter 6 it is also demonstrated that the EOA approach frequently leads to (often) undesirable singular solutions and to prevent this the use of well variable transformation methods and, in particular, inversion and the bilinear transform is investigated.

Chapter 7 extends the basic results of the previous chapters are extended to include:

1. nD (n > 2) systems,
2. linear repetitive processes (a distinct class of 2D linear systems of both theoretical and applications interest), and
3. causal systems described by Laurent polynomials.

Finally, Chapter 8 gives a critical and comparative overview of the progress reported in this monograph and give directions for future research.

Where appropriate, the algorithms and procedures in this monograph have been illustrated by examples which make extensive use of symbolic computing and, in particular, Maple.

I am grateful to Prof. Marian S. Piekarski (Technical University of Wrocław, Poland), who introduced me to the subject of nD linear systems and encouraged me to work in this area. Special thanks are due to Prof. T. Kaczorek (Warsaw University of Technology, Poland), whose encouragement and advice over many years has been of critical importance to my work. Also I am deeply thankful to Prof Eric Rogers (University of Southampton, UK) and to Professor David Owens (University of Sheffield, UK) for many fruitful discussions, research collaboration on, in particular, repetitive processes and help in preparing the final manuscript. Similar thanks are also due to Prof. Nirmal K. Bose (Pennsylvania State University, USA). Finally, I would like to thank thank Prof. Józef Korbicz (Technical University of Zielona Góra) the head of the research group where I currently work for encouragement to proceed with this monograph and my colleague Dr. Dariusz Ucinski for his excellent proof-reading.

Last but not least, I would like to thank my parents, my wife Eva and children Philip and Agatha for their patience, understanding and support.

Zielona Góra, September 2000 *Krzysztof Gałkowski*

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Notation

R	set of real numbers
N	set of natural numbers
$\{a,b,\dots\}$	set of elements a, b, ...
$\varphi : X \to Y$	the operator from X to Y
R^n	real n-dimensional space
$R[s_1, s_2]$	the ring of bivariate polynomials in s_1 and s_2
s_i, s, z_i, z	complex variables
$\oplus$	direct sum
$\in$	element inclusion
$\subset$	set inclusion
$\sum$	sum
$\mathrm{Mat}_{m,n}(X)$	the set of all m by n matrices with elements from X
A^T	the transpose of the matrix A
A^{-1}	the inverse of the matrix A
$\mathbf{0}$	zero matrix
$\mathbf{0}_{mn}$	zero m by n matrix
I	unit matrix
I_m	unit m by m matrix
$\deg_s[a]$	degree of the polynomial a in the variable s
rank A	the rank of the matrix A
Augment	the operation of the matrix size augmentation
BlockAugment	block operation Augment
L	the left (row) elementary operation
R	the right (column) elementary operation
L	the block left (row) elementary operation
R	the block right (column) elementary operation