Propositional and Predicate Calculus: A Model of Argument

Derek Goldrei

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A Model of Argument

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PREFACE

How to Use This Book

This book is intended to be used by you for independent study, with no other reading or lectures etc., much along the lines of standard Open University materials. There are plenty of exercises within the text which we would recommend you to attempt at that stage of your work. Almost all are intended to be reasonably straightforward on the basis of what's come before and many are accompanied by solutions – it's worth reading these solutions as they often contain further teaching, but do try the exercises first without peeking, to help you to engage with the material. Those exercises without solutions might well be very suitable for any tutor to whom you have access to use as the basis for any continuous assessment of this material, to help you check that you are making reasonable progress. But beware! Some of the exercises pose questions for which there is not always a clear-cut answer: these are intended to provoke debate! In addition there are further exercises located at the end of most sections. These vary from further routine practice to rather hard problems: it's well worth reading through these exercises, even if you don't attempt them, as they often give an idea of some important ideas or results not in the earlier text. Again your tutor, if you have one, can guide you through these.

The book is also peppered with notes in the margins, like this! They consist of comments meant to be on the fringe of the main text, rather than the core of the teaching, for instance reminders about ideas from earlier in the book or particularly subjective opinions of the author.

If you would like any further reading in logic textbooks, there are plenty of good books available which use essentially the same system, for instance those by Enderton [12], Hamilton [18], Mendelson [25] and Cori and Lascar [7].

Acknowledgments

I would like to thank all those who have in some way helped me to write this book. My enthusiasm for the subject was fuelled by Robin Gandy, Paul Bacsich, Jane Bridge, Angus Macintyre and Harold Simmons, when I studied at the Universities of Oxford and Aberdeen. Anything worthwhile I have successfully learnt about teaching stems from my colleagues at the Open University and the network of mathematicians throughout the UK who support the Open University by working as Associate Lecturers, external assessors and examiners. They have taught me so much. It has been particularly stimulating writing this book alongside producing the Open University's course on Mathematical Logic (with a very different angle on the subject) with Alan Pears, Alan Slomson, Alex Wilkie, Mary Jones, Roger Lowry, Jeff Paris and Frances Williams. And it is a privilege to be part of a university which puts so much care and effort into its teaching and the support of its students. The practicalities of producing this book owe much to my publishers, Stephanie Harding and Karen Borthwick at Springer; and to my colleagues at the Open University who have done so much to provide me with a robust and attractive LaTeX system: Alison Cadle, David Clover, Jonathan Fine, Bob Margolis and Chris Rowley. And thanks to Springer, I have received much invaluable advice on content from their copy-editor Stuart Gale and their anonymous,

Plainly the blame for any errors and inadequacies of this book lies entirely with me. But perhaps at some deep and subtle level, the fault lies with everyone else!

very collegial, reviewers. I would also like to thank Michael Goldrei for his work on the cover design.

Perhaps the main inspiration for writing the book is the enthusiasm and talent for mathematical logic displayed by my old students at the Open University and at the University of Oxford, especially those of Somerville, St. Hugh's and Mansfield Colleges. In particular I'd like to thank the following for their comments on parts of the book: Dimitris Azanias, David Blower, Duncan Blythe, Rosa Clements, Rhodri Davies, David Elston, Michael Hopley, Gerrard Jones, Eleni Kanellopoulou, Jakob Macke, Zelin Ozturk, Nicholas Thapen, Matt Towers, Chris Wall, Garth Wilkinson, Rufus Willett and especially Margaret Thomas.

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