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Revenue Management and Pricing Analytics

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To my wife, Mengqian.

Guillermo Gallego

*To my parents, Gülşen and Ali Topalođlu,
for their unconditional love and support.*

Huseyin Topaloglu

Foreword

When Kalyan Talluri and I began to work on our book *The Theory and Practice of Revenue Management* (Talluri and van Ryzin 2004b) around the millennial year of 2000, the task was as clear-cut as it was daunting, attempting for the first time to synthesize a field that was young yet highly impactful, spanned multiple scientific disciplines, and whose core contributions lay scattered throughout both the academic and industrial communities.

The boundaries of what was and was not revenue management were unclear. Basic concepts, terminology, and notation were muddled. The audience for the work was ambiguous. Was it for scientists or business people? Researchers or practitioners? Students or experienced professionals? And even the format was not entirely clear: textbook, monograph, or reference book?

The end result published in 2004 was an imperfect compromise. It was overly expansive in places and not deep enough in others. Parts were accessible to general readers while others only to postdoctoral researchers. Thanks to our naivete in dealing with academic publishers, it was a pricing book that was embarrassingly overpriced.

Still, it managed to serve its purpose of defining the field for what it was: a productive blend of scientific theory with industry practice, enabled by a creative merging of business, engineering, and economic mindsets. It cataloged both the business context that leads to the practice of revenue management and the fundamental mathematical models and methods behind that practice. It helped expand the field, serving as an entry point for both students and experienced professionals alike. In these ways, it accomplished our goals in writing it.

However, 15 years have passed since *The Theory and Practice of Revenue Management* was published. Research and practice have exploded since then. Entirely new industries like ridesharing have emerged where dynamic pricing is central to the entire service model. And the need for an updated book on the field has been growing.

Revenue Management and Pricing Analytics by Guillermo Gallego and Huseyin Topaloglu is that book—and more. For starters, Gallego and Topaloglu have done more to advance the state of the art in revenue management over the past 15 years

than any other researchers in the field, so the reader is in expert hands. Both are meticulous mathematicians too. And the book is clearly focused on three core topic areas central to modern revenue management: (1) traditional revenue management (capacity control and overbooking), (2) choice-based models and assortment optimization, and (3) pricing analytics (price-based controls).

Along with this increased focus comes increased depth. In particular, in the section on traditional capacity controls, the authors present in detail many new methods and results on these traditional problems, including performance bounds, new approximate dynamic programming methods, and probabilistic admission methods. The extensive section on choice-based revenue management and assortment optimization is almost entirely new, with many recent results on, for example, Markov chain choice models and the authors' signature results on the sales-based linear programming model. This section will also prove valuable to the many applied scientists working on retailing and recommendation system problems which involve assortment optimization. Lastly, the section on pricing analytics contains an updated treatment of classical price-based revenue management problems and its impact on consumers, new results on the learning-earning (explore-exploit) trade-off in combining pricing estimation and optimization, as well as results on competitive pricing and assortment optimization.

In addition to the updated and greatly expanded coverage of these core revenue management areas, the book provides an extensive collection of problems at the end of each chapter. It therefore serves as an invaluable resource for educating the next generation of revenue management scientists.

In short, it has all the qualities one could hope for as the new standard reference in the field of revenue management.

Lastly, on a personal note, I can think of no two colleagues I would rather see write this book. Guillermo has been a close friend since we carpooled together as assistant professors at Columbia and wrote our first joint paper on dynamic pricing. That paper was the start of both our careers in the field. Huseyin and I are currently colleagues together at Cornell Tech, and before that, I watched with admiration as his research career has blossomed over the past decade.

It makes me smile to see what they have accomplished together.

Cornell Tech, New York, NY, USA
Lyft Inc., San Francisco, CA, USA
March 2019

Garrett van Ryzin

Preface

Revenue management can be defined as a data-driven, computerized system to support the tactical pricing of perishable assets at the micro-market level to maximize expected revenues from sunk investments in capacity. More broadly, revenue management includes product design at the strategic level (e.g., coming up with travel restrictions and ancillary services that comprise an airline fare class as well as upgrade and upsell policies to be used) and inventory control of fare classes at the operational level (e.g., opening and closing fare classes in response to remaining capacities). While inventory control for the fare classes is the main lever in revenue management, dynamic pricing is about selecting optimal price paths over the sales horizon. Pricing analytics goes beyond dynamic pricing to include the study of consumer surplus and welfare and issues related to competition and on-line learning.

Revenue management originated in the United States after the deregulation of the airline industry in the late 1970s. The often-heard story of its origins is the dilemma faced by American Airlines (AA) when People Express (PE) offered deeply discounted fares in markets that were crucial to AA, resulting in unsustainable capacity spoilage. The alternative of matching the fares of PE was not feasible for AA due to its more expensive cost structure. The solution AA came up with was to offer non-refundable super-saver fares designed to undercut PE, while restricting availability to protect inventory for consumers willing to pay full fares. The saga ends a few months later with the bankruptcy of PE. While competition drove AA to a market segmentation strategy, competition is not essential for segmentation to be profitable. Indeed, market segmentation policies have existed for centuries in monopolies, ranging from canal and river tolls in China's Ming dynasty to Danish Sound tolls in the sixteenth century. What was new in the AA-PE saga were the restrictions (e.g., time of sale and booking limits) that went into the design of the super-saver fares, which made these fares available in limited quantities and to consumers who were willing to book tickets in advance. Interestingly, the use of different fare classes can improve expected revenues even if there is only one market segment.

Revenue management and dynamic pricing are essential tools for firms in the travel and leisure industry including airlines, hotels, car rental companies, art performances, and sports events. Revenue management systems were developed by some airlines, while others acquired customizable systems from vendors. A similar development has occurred for the hotel industry, car rental companies, and other segments of the leisure industry.

Applications of revenue management and dynamic pricing have expanded to other areas including e-commerce, health care, media, telecom, and financial services. In e-commerce, for example, retailers have to decide which products to offer and how to display them on their webpages. These are assortment optimization and product framing problems that require a deep understanding of consumer choice models. Firms offering new products with a short life cycle need to simultaneously learn and earn during the product sales horizon. Many of these new applications require modern revenue management and pricing analytics that go beyond the models developed in the travel and leisure industries.

There are several books in revenue management available to the reader. Here, we briefly mention three books and explain how the book in your hands (or on your screen) is different. *The Theory and Practice of Revenue Management* (Talluri and van Ryzin 2004b) has a comprehensive treatment of the subject that artfully mixes theory and practice including chapters on demand estimation and the economics of pricing. The book is aimed at a core audience with at least a master's degree or higher in engineering, operations research, statistics, or economics. The scope of our book is not as broad as Talluri and van Ryzin (2004b), as it does not cover demand estimation, a subject for which there are many excellent references. While also a technical book, our book aims at a broader audience that includes advance undergraduates in operations research, statistics, economics, and other related fields. Our book provides an up-to-date treatment of the subject containing many results that were not available in 2004.

The book *Pricing and Revenue Optimization* (Phillips 2005) provides a high-level treatment of the subject with many interesting managerial insights. It is the textbook of choice for courses in business schools, executives, and practitioners that are less analytically tolerant. It remains relevant today for both technical and nontechnical readers. Our book is almost orthogonal to Phillips (2005), as the focus is not on making the concepts accessible to executive or nontechnical readers.

Lastly, the book *The Oxford Handbook of Pricing Management* (Ozer and Phillips 2012) focuses on showcasing the recent research in revenue management, along with a detailed discussion of the industries in which revenue management tools find applications. The book is a comprehensive edited volume with contributed chapters. As such, it does not lend itself easily to classroom use at the master's or introductory PhD level. Our book is intended to be useful as a textbook.

The book is divided into three parts: traditional revenue management, revenue management under customer choice, and pricing analytics. Each part is approximately of the same length and written in a self-contained way, so readers can read them independently, although reading the first part may make the second part easier

to understand. Each chapter ends with bibliographical notes where the reader can find the sources of the material covered as well as many useful references. Proofs of some important technical results can be found in the appendix of each chapter. Solving the end-of-chapter problems helps reinforce the material in the book, with some of the questions expanding on the subject.

The first part of the book, on traditional revenue management, has a rigorous treatment of the most important models developed in industry where time is treated implicitly and those developed in academia where time is considered explicitly. Coverage includes Littlewood's rule, bounds and heuristics for single resource and network models, overbooking, and some extensions such as the generalized newsvendor problem with convex costs. Academic models treat time explicitly which allows for arbitrary fare arrival sequences. Heuristics are developed, and conditions under which simple heuristics are asymptotically optimal are discussed.

The second part of the book, on revenue management under customer choice, is a modern treatment of revenue management that largely gives up on Littlewood's rule and its extensions in favor of a dynamic programming framework based on discrete choice models and the machinery of assortment optimization. Bounds, heuristics, and asymptotic properties are presented. This section starts with a chapter on discrete choice models, followed by a chapter on assortment optimization as these are the foundation of revenue management models under customer choice. A large class of discrete choice models can be approximated by the Markov chain choice model, and this model can in turn be used to develop bounds and heuristics for the network revenue management model that have the same complexity as the models for the independent demand model.

The third part of the book is on pricing analytics. It starts with an in-depth treatment of pricing models with linear costs (that arise as dual prices of capacity constraints) and a rigorous treatment of the necessary and sufficient conditions for existence and uniqueness of optimal prices. We show that firms prefer input costs that are random and that consumers also prefer random prices. We establish conditions on the pass-through rate under which randomness in inputs for the firm benefits both the firm and the consumer. We also provide treatments of a number of extensions including call options on capacity and models that allow for bargaining and concise treatments of the peak-load pricing and priority pricing. The chapter on dynamic pricing extends known formulations to allow for inventory replenishments, holding costs, discounted costs, compound Poisson demands, and dynamic non-linear pricing. The chapter on online learning presents some of the most relevant results in learning and earning, while the final chapter on competition presents models relevant to revenue management and dynamic pricing.

There is enough material in the book for a full-semester course for advanced undergraduate or master's students. Parts I and II can be covered in about 9 weeks and Part III in about 4 weeks excluding the last two chapters on online learning and competition, which can be assigned as independent readings.

The book has also been used for PhD seminars on revenue management and pricing analytics. These seminars would meet once a week for 3 hours at a time. Graduate students were asked to read the chapters in advance, with the instructor covering the material at a high level in 6 weeks and with students presenting papers during the next 4 weeks, leaving the last 3 weeks for project presentations.

Clearwater Bay, Hong Kong
New York, NY, USA
January 2019

Guillermo Gallego
Huseyin Topaloglu

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