Part III
Continuous Time Processes

The next part of the book is concerned with the development of useful representations for continuous time models and their application. In the setting of discrete time one could obtain very useful representations using the chain rule. This is no longer the case in continuous time, but it is possible to obtain representations of a form analogous to those in discrete time using the Radon–Nikodym Theorem on path space for one inequality and stochastic control arguments for the reverse inequality. The representations for infinite dimensional Brownian motion and Poisson random measures are the topic of Chap. 8.

For many models in continuous time, driving noises enter in a more regular fashion than for their discrete time counterparts. In particular, for stochastic ordinary and partial differential equations driven by Brownian and Poisson noise, these noises enter in an “affine” manner, which is meant to include both what is often referred to as “additive” noise (constant noise coefficient), and “multiplicative” noise (state-dependent noise coefficient). As a consequence, the mapping from the noise space into the state space often has more structure and regularity than in the corresponding discrete time setting. One can compare, for example, Markov processes described by SDEs with Markov processes of the type studied in Chap. 4. A consequence of the improved regularity is that one can identify conditions on the map that takes the noise process into the state process that are sufficient for large and moderate deviation principles to hold, and which are broadly applicable. This is the topic of Chap. 9, which proves large and moderate deviation properties for general mappings of Brownian motion and Poisson random measures.

The results of Chap. 9 are specialized to prove large and moderate deviation properties for finite dimensional systems in Chap. 10. The setting is that of a standard SDE model with regular (e.g., Lipschitz continuous) coefficients. Infinite dimensional systems driven by Brownian noise, including SPDE, are the topic of Chap. 11. Our previously published versions of these results have found wide use in small noise SPDE models with multiplicative noise, and a listing of some of the applications is given in the notes at the end of Chap. 9.

Chapter 12 presents another use of the representation for infinite dimensional Brownian motion, which is to the large deviation theory for small noise flows of diffeomorphisms. Also included is an application to Bayesian image reconstruction. Chapter 13 returns to finite dimensional models and considers those with special features, and emphasizes problems with less regularity in the mapping that takes the noise into the state. Chapter 13 illustrates a useful aspect of the weak convergence approach, which is that by converting large deviation questions into questions of weak convergence, one can more easily understand exactly what regularity conditions are really needed. For related works that highlight this same feature, see [1, 2, 3]. An important class of problems covered in Chap. 13 is that of large and moderate deviations for certain pure jump processes that, when written in the form of a stochastic differential equation driven by a Poisson random measure, have discontinuous coefficients in the stochastic integral and are therefore not covered by Chap. 10.