# Part I
# Laplace Principle, Relative Entropy, and Elementary Examples

The intent of this book is to explain how variational representations and weak convergence methods can be used for the qualitative and quantitative analysis of rare events (large deviation theory), and to address related questions of numerical analysis (accelerated Monte Carlo). This introductory part consists of three chapters. In the first chapter, the equivalence between a large deviation principle and the corresponding Laplace principle is demonstrated for random variables that take values in a Polish space. The Laplace principle is a "bounded and continuous test function" characterization, and it asserts the convergence of normalized logarithms of certain exponential integrals. With this exponential integral characterization in hand, the rest of the chapter proves a number of general results in the theory.

Chapter 2 discusses relative entropy and its many attractive properties. Relative entropy plays a central role in everything that is done in the book, owing to its appearance in a fundamental variational formula for the exponential integrals. In particular, the chain rule of relative entropy, which is the key to obtaining useful representations for processes with structure (e.g., Markov processes) is stated and proved. The chapter also proves or references various results on tightness of probability measures that will be used in the weak convergence analysis.

Chapter 3 shows how refined variational representations can be combined with weak convergence arguments to prove the large deviation principles for some basic models: Sanov's theorem and Cramér's theorem, and stochastic differential equations driven by Brownian and Poisson noise (the latter using representations for continuous time processes that will be proved in Chap. 8). Although the analysis of models that will be considered in later chapters requires considerably more detail, all the main ideas concerning how the representations should be used can be seen in these simple examples.