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Dynamic Markov Bridges and Market Microstructure

Theory and Applications

Springer

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To our families
Emel, Christian and Alice.

Preface

During the course of our research on equilibrium models of asymmetric information in market microstructure theory, we have realised that one needed to apply techniques from different branches of stochastic analysis to treat these models with mathematical rigour. However, these subfields of stochastic analysis—to the best of our knowledge—are not presented in a single volume. This book intends to address this issue and provides one concise account of all fundamental theory that is necessary for studying such equilibrium models.

Equilibrium in these models can be viewed as an outcome of a game among asymmetrically informed agents. The less informed agents in these games endeavour to infer the information possessed by the agents with superior information. This obviously necessitates a good understanding of the stochastic filtering theory. On the other hand, the equilibrium strategy of an agent with superior information is to drive a commonly observed process to a given random variable without distorting the unconditional law of the process. Construction of such strategy turns out to be closely linked to the conditioning of Markov processes on their terminal value. Moreover, this construction needs to be admissible and adapted to the agent's filtration, which brings us to the study of stochastic differential equations (SDEs) representing Markov bridges. Therefore, an adequate knowledge of stochastic filtering, Markov bridges and SDEs is essential for a thorough analysis of asymmetric information models.

The aim of this book is to build this knowledge. Although there are many excellent texts covering various aspects of the aforementioned three fields, the standard assumptions in these literature are often too restrictive to be applied in the context of asymmetric information models. Driven by this need from applications we extend a lot of results known in the literature. Therefore, this book can also be viewed as a complementary text to the standard literature. Proofs of statements that already exist in the literature are often omitted and a precise reference is given.

This book assumes the reader has some knowledge of stochastic calculus and martingale theory in continuous time. Although familiarity with SDEs will make its reading more enjoyable, no prior knowledge on this subject is necessary. The exposition is largely self-contained, which allows it to be used as a graduate textbook on equilibrium models of insider trading.

The material presented here is divided into two parts. Part I develops the mathematical foundations for SDEs, static and dynamic Markov bridges, and stochastic filtering. Equilibrium models of insider trading and their analysis constitute the contents of Part II.

In Chap. 1 we present preliminaries of the theory of Markov process including the strong Markov property and the right continuity of the filtrations and introduce Feller processes. Naturally in this chapter we select the results that are necessary for the development of the theory of Markov bridges.

As proofs of the results presented in Chap. 1 will remain unaltered under an assumption of path continuity, we have refrained from assuming path regularity in that chapter. However, we will confine ourselves to diffusion processes for the rest of the book since the theory of SDE representation for general jump-diffusion bridges is yet to be developed.

Chapter 2 is devoted to stochastic differential equations and their relation with the local martingale problem. In particular standard results on the solutions of SDEs and comparison of one-dimensional SDEs have been extended to accommodate the exploding nature of the coefficients that are inherent to the SDEs associated with bridges.

Chapter 3 is an overview of stochastic filtering theory. Kushner–Stratonovich equation for the conditional density of the unobserved signal is introduced and uniqueness of its solution is proved using a suitable filtered martingale problem pioneered by Kurtz and Ocone [84].

Using the theory presented in Chaps. 1 and 2 we develop the SDE representation of Markov processes that are conditioned to have a prespecified distribution μ at a given time T in Chap. 4. Two types of conditioning are considered: weak conditioning refers to the case when μ is absolutely continuous with respect to the original law at time T whereas strong conditioning corresponds to the case when μ is a Dirac mass. We also discuss the relationship between such bridges and the enlargement of filtrations theory.

The bridges constructed in Chap. 4 are called static since their final bridge point is given in advance. Chapter 5 considers an extension of this theory when the final point is not known in advance but is revealed over time via the observation of a given process. To verify that the law of these dynamic bridges coincides with the law of the original Markov process when considered in their own filtration, we use techniques from Chap. 3.

Part II is concerned with the applications of the theory in Part I and starts with Chap. 6, which provides the description of the Kyle–Back model of insider trading as the underlying framework for the study of equilibrium in the chapters that follow. Chapter 6 also contains a proof in a general setting of the ‘folk result’ that one can limit insider’s trading strategies to absolutely continuous ones. Chapter 7 presents an equilibrium in this framework when the inside information is dynamic in the absence of default risk. It also shows that equilibrium is not unique in this family of models. Chapter 8 studies the impact of default risk in the equilibrium outcome.

The book grew out of a series of paper with our long-term collaborator and colleague Luciano Campi, who has also read large portions of the first draft and suggested many corrections and improvements for which we are grateful. We also thank Christoph Czichowsky and Michail Zervos for their suggestion on various parts of the manuscript. This book was discussed at the Financial Mathematics Reading Group at the LSE and we are grateful to its participants for their input.

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October 2017

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Contents

Frequently Used Notation

A, A_t	Generator of a Markov process, see Sect. 2.1
$\mathscr{B}_t$, $\mathscr{B}_t^-$	For a given $T < \infty$, $\mathscr{B}_t$ is the history of the coordinate process on $C([0, T], \mathbf{E})$ up to time-t. $\mathscr{B}_t^-$ is the history of the coordinate process on $C([0, T), \mathbf{E})$ up to time-t. See the discussion preceding Theorem 4.2
$b\mathscr{G}$	The set of bounded and $\mathscr{G}$-measurable functions
$C(U, V)$, $\mathbb{C}(U)$, $\mathbb{C}$	$C(U, V)$ is the space of V-valued continuous functions on U, $\mathbb{C}(U) = C(U, \mathbb{R})$, $\mathbb{C} = \mathbb{C}(\mathbf{E}_\Delta)$. See also Remark 1.3 for the relationship between different subspaces of continuous functions
$\mathbb{C}_0(\mathbf{E})$, $\mathbb{C}_0$	$\mathbb{C}_0(\mathbf{E})$ is the space of $\mathbb{R}$-valued continuous functions on $\mathbf{E}$ vanishing at infinity, $\mathbb{C}_0 = \mathbb{C}_0(\mathbf{E}_\Delta)$
$\mathbb{C}_K(\mathbf{E})$, $\mathbb{C}_K$	$\mathbb{C}_K(\mathbf{E})$ is the space of $\mathbb{R}$-valued continuous functions on $\mathbf{E}$ with compact support, $\mathbb{C}_K = \mathbb{C}_K(\mathbf{E}_\Delta)$
$\mathbb{C}^n(\mathbf{E})$, $\mathbb{C}^n$	$\mathbb{C}^n(\mathbf{E})$ is the space of $\mathbb{R}$-valued, n-times continuously differentiable functions on $\mathbf{E}$, $\mathbb{C}^n = \mathbb{C}^n(\mathbf{E}_\Delta)$
$\mathbf{E}$, $\mathbf{E}_\Delta$, $\mathscr{E}$	$\mathbf{E}$ is locally compact separable metric space, $\mathbf{E}_\Delta$ is one-point compactification of $\mathbf{E}$, and $\mathscr{E}$ is the Borel sigma-algebra on $\mathbf{E}$
E^x	Expectation operator corresponding to P^x
ε_x	Measure with point mass at x
$\mathscr{F}_t^0$, $\mathscr{F}_t'$, $\mathscr{F}^0$	$\mathscr{F}_t^0$ is the history of the underlying Markov process up to time-t, $\mathscr{F}_t'$ is its future evolution after time-t, and $\mathscr{F}^0 = \sigma(\mathscr{F}_t^0, \mathscr{F}_t')$
M_t^f	See the expression in (2.2)
$^\circ\eta$	Optional projection of η, see Definition 3.1

P^{μ}, P^{x}	For a given Markov process X with initial distribution μ, P^{μ} is the probability measure on $\mathscr{F}^0$ generated by X. $P^x = P^{\varepsilon_x}$. See Eq. (1.4) in this respect
π_t, $\pi_t f$	Conditional distribution of the signal. See Sects. 3.1 and 3.2
$P_t f$, $P_{s,t} f$	See Eq. (1.7) and Assumption 4.2 correspondingly
$P_{s,t}(\cdot, \cdot)$	Markov transition function, see Definition 1.2
θ	Shift operator, see Eq. (1.6)
U^{α}	α-potential operators, see Definition 1.5