Springer Undergraduate Texts in Mathematics and Technology

More information about this series at http://www.springer.com/series/7438

Arlie O. Petters • Xiaoying Dong

An Introduction to Mathematical Finance with Applications

Understanding and Building Financial Intuition

Springer

Arlie O. Petters
Department of Mathematics
Duke University
Durham, NC, USA

Xiaoying Dong
Department of Mathematics
Duke University
Durham, NC, USA

ISSN 1867-5506 ISSN 1867-5514 (electronic)
Springer Undergraduate Texts in Mathematics and Technology
ISBN 978-1-4939-3781-3 ISBN 978-1-4939-3783-7 (eBook)
DOI 10.1007/978-1-4939-3783-7

Library of Congress Control Number: 2016939449

Mathematics Subject Classification (2010): 91Gxx

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG Switzerland

To my loving wife, Elizabeth Petters, for being at my side unconditionally and my child, Preston Petters, who inspires me with his intense curiosity.

A.O. Petters

To my dear husband and best friend, Xin Zhou. I could not imagine to complete my part of the contribution to this book without his love and support.

X. Dong

Preface

Rationale and Aim

Given the increasing intricacies and interconnectedness of financial firms' activities and the potential opportunities and risks to which they expose themselves and the world's economy, the next generation of financial engineers needs to master an extensive array of mathematical financial models. Indeed, one of the current challenges in finance is that the complexity of modern securities and markets has forced modelers to employ increasingly sophisticated mathematical tools to address financial issues, creating a widening gap between the qualitative and quantitative approaches to finance.

Our book seeks to address this gap by introducing the quantitative aspects of finance to students with either a qualitative background or no background in the subject. At a firm the traders, risk managers, etc. employ proprietary analytical and numerical models custom made to the needs of their firm. However, since open access to such models is prohibited, the book instead strives to give students a fundamental understanding of key financial ideas and tools that form the basis for building more realistic models, including those of a proprietary nature.

Distinctive Features and Benefits

This book is distinct in how it emphasizes and pedagogically conveys in an accessible manner the theoretical understanding and applications of the mathematical models forming key pillars of modern finance.

First, the book keeps a good balance between mathematical derivation and description for the sake of providing an adequate level of rigor and depth in mathematics and maintaining accessibility to the reader, which in turn adds flexibility of material selection for the instructor (e.g., Chapter 7 may be taught earlier). Specifically, this book addresses the gap between textbooks that offer a theoretical treatment without many applications and those that simply present and apply formulas without appropriately deriving them. Indeed, theoretical understanding is incomplete without enough practice in applications, and applications are risky without a rigorous theoretical understanding. To accomplish this, the book contains numerous carefully chosen examples and exercises that reinforce a student's conceptual understanding and develop a facility with applications. Indeed, the exercises are divided into conceptual, application, and theoretical problems that probe the material deeper.

Second, beyond a few required undergraduate mathematics courses (see Prerequisites below), this book is essentially self-contained. The large number of necessary financial terminologies and concepts can be overwhelming to a student new to finance. For this reason, after introducing some central, big-picture financial ideas in the first chapter, we present the financial minutia along the way as needed. We have tried to make the book self-contained in this regard through thoughtfully chosen illustrative applications starting at the ground level with simple interest. *We then gradually increase the difficulty as the book develops, ranging across compound interest, annuities, portfolio theory, capital market theory, portfolio risk measures, the role of linear factor models in portfolio risk attribution, binomial tree models, stochastic calculus, derivatives, the martingale approach to derivative pricing, the Black-Scholes-Merton model, and the Merton jump-diffusion model.*

Third, the book is also useful for students preparing either for higher level study in mathematical finance or for a career in actuarial science. For example, the syllabi for the actuarial Financial Mathematics Exam (Exam 2/FM) and Models of Financial Economics Exam (Exam 3F/MFE) include many topics covered in the book.

Prerequisites

The required mathematics consists of introductory courses on multivariable calculus, probability, and linear algebra. Along the way, we introduce additional mathematical tools as needed—e.g., some measure theory is presented from scratch.

No background in finance is assumed. As noted above, the necessary financial concepts and tools are introduced in the text, with the first chapter giving an overview of several common finance terminologies associated with securities and securities markets.

Our book does *not* require computer programming. In our experience, finance courses based on computer programming are best taken after students have developed a fundamental understanding of the theoretical architecture of financial models.

Audience

The text is aimed at *advanced undergraduates* and *master's degree students* who are either new to finance or want a more rigorous treatment of the mathematical models used in finance. The students typically are from economics, mathematics, engineering, physics, and computer science.

We also believe that a faculty member who is teaching finance for the first time will find this introduction readily manageable. Professionals working in finance who would like a refresher or even clarification on some of the theoretical and conceptual aspects of mathematical finance will benefit from the text.

Scope and Guide

The chapters are organized naturally into four parts and range over the following topics:

- Part I (Chapters 1 and 2):
  introduction to securities markets and the time value of money
- Part II (Chapters 3 and 4):
  Markowitz portfolio theory, capital market theory, and portfolio risk measures
- Part III (Chapters 5 and 6):
  modeling underlying securities using binomial trees and stochastic calculus
- Part IV (Chapters 7 and 8):
  derivative securities, BSM model, and Merton jump-diffusion model

The material was tested in courses offered to upper-level undergraduates and master's degree students. Below are two examples of possible topics that may serve as a guide for **semester-long courses**:

- *Introduction to Mathematical Finance*: securities markets (Chapter 1), the time value of money (Chapter 2), Markowitz portfolio theory, capital market theory, and portfolio risk measures (Chapters 3–4), binomial security pricing (Chapter 5, omit most derivations), Itô's formula and geometric Brownian motion (Sections 6.8 and 6.9), forwards, futures, and options (Sections 7.2, 7.3, and 7.5), and call option pricing with applications (Sections 8.3, 8.2.2, 8.5, and 8.6.2).
- *Introduction to Financial Derivatives*: modeling underliers in discrete time (Sections 5.1–5.3), stochastic calculus and modeling underliers in continuous time (Section 5.4 and Chapter 6), general aspects of forwards, futures, swaps, and options, including trading strategies (Chapters 7), the Black-Scholes-Merton (BSM) model, BSM p.d.e. approach to pricing European-style options, risk-neutral approach to pricing European-style options, applications to warrants, delta hedging, managing portfolio risk, and extension of the BSM model to the Merton jump-diffusion model (Chapter 8).

A **year-long course** on introductory mathematical finance can be based on the entire book. The book can also be used as a reference for students enrolled in a mathematical finance **independent study course**.

Acknowledgments

Specials thanks to the following individuals for their feedback and assistance:

Daniel Aarhus
Amir Aazami
Stanley Absher
Vibhav Agarwal
Hengjie Ai
Mitesh Amarthaluru
Vlad Bouchouev
Michael Brandt
Esteban Chavez
Rui Chen
Kyuwon Choi
Qian Deng
Christian Drappi
Zachary Freeman
Tingran Gao
William Grisaitis
Xiaosheng Guo
Zhonglin Han
John Hyde
Huseyin Kortmaz
Baolei Li
Junchi Li
Li Li
Nan Li
Qiao Li
Li Liang
Lu Liu
Ruisi Ma
Tanya Mallavarapu
Xavier Mela
Vadim Mokhnatkin
Julia Ni
James Nolen
Vivek Oberoi
Feng Pan
Chloe Peng
Hal Press
Hui Qi
Zhaozhen Qian
Hayagreev Ramesh
Emma Rasiel
Tianhua Ren
Chelsea Richwine
Irving Salvatierra
Andrew Schretter
Yuhang Si
John Sias
Maxwell Stern
Lingran Sun
Alberto Teguia
Nicholas Tenev
Dominick Totino
Chi Trinh
Dan Turtel
Kari Vaughn
Robert Vanderbei
Kevin Wan
Chenyu Wang
David Williams
Chao Xu
Hangjun Xu
Lu Xu
Junkai Xue
Chao Yang
Jiahui Yang
Ashley Yeager
Jeong Yoo
Yanchi Yu
Yunliang Yu
Javier Zapata
Xiaodong Zhai
Biyuan Zhang
Yang Zhang
Bowen Zhao
Ruiyang Zhao
Xiaoyang Zhuang
Zilong Zou

We are also thankful to *Elizabeth Loew* of Springer for her support and guidance along the entire way and to *Lisa Goldberg* for her valuable comments and constructive suggestions. AP is indebted to Duke University for providing the financial support needed to hire many students who assisted with writing computer codes, checking calculations, etc. He is also extremely grateful to his wife, *Elizabeth Petters*, for her patience, love, and steadfast encouragement throughout the project. XD would like to express her gratitude to her husband *Xin Zhou* who saw her through this book and offered great suggestions.

Durham, NC, USA A.O. Petters
Durham, NC, USA X. Dong
2016

Contents