Part I
Theory

After the description of an evolution phenomenon by a mathematical model, the next step in the study of the dynamics is to set the mathematical framework, which inserts the specific instance into a more abstract context. The collection of definitions and theorems traces the "boundaries" of the theory: it allows to determine the applicability and the features of the mathematical model, to lay the foundation for the construction of numerical models and, finally, to validate the results of the numerical simulations. The purpose of this theoretical part is to introduce the basic notation and to present some background material, required throughout the book. Nowadays, the list of books dealing with theory and applications of DDEs is quite long [12, 20, 22, 70, 72, 81, 91, 93, 121, 122, 126, 147, 151, 176]. In particular, we mainly follow [70, 91, 121], where the interested reader can find further details.

Before studying the dynamics, one wants first to be sure that the mathematical problem is well-posed, i.e., there exists a unique solution, which depends continuously on the data (original definition in [89]). Therefore, in the first chapter, after setting the basic notation, we recall some classic results on the solvability of the Cauchy problem for DDEs. In the applications, it is important to identify some particular solutions, such as, e.g., the equilibria, and to predict the effect of small perturbations. This concerns the stability theory and the corresponding definitions are given in Sect. 2.3. By the Principle of Linearized Stability, the stability analysis of any solution of interest of a nonlinear equation can be attributed to the behavior of the corresponding linearized DDE. In this context, the understanding of the linear case is crucial, adding a further motivation for its study. In fact, as already pointed out in the introductory examples of Chap. 1, linear DDEs arise when modeling different real-life linear phenomena or by linearization of nonlinear ones.

The focus is on linear DDEs of autonomous and periodic type, whose necessary theory is summarized in Chaps. 3 and 4, respectively. Such classes of linear equations are of great interest in applications and the stability theory is well established. We remark that the linearization of nonlinear autonomous DDEs at equilibria and periodic solutions leads, respectively, to linear autonomous DDEs and linear periodic DDEs.

For completeness, we recall that the method of Lyapunov functionals has been successfully applied to examine the stability of solutions of nonlinear DDEs [91, 121, 122]. The approach furnishes sufficient conditions for stability and instability, generalizing the method of Lyapunov for ODEs.