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Basic Real Analysis

Second Edition

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To Shohreh, Mahsa, and Zubin

Preface

Ah, Love! could thou and I with Fate conspire
To grasp this sorry Scheme of Things entire,
Would not we shatter it to bits—and then
Re-mould it nearer to the Heart's Desire!

Omar Khayyam, Rubaiyat

More than 10 years have passed since the publication of the first edition of this textbook. During these years, a large number of monographs dealing with the same topics have appeared. Some of them have been included in the new bibliography. In addition, a wealth of material is now freely available online, some of it posted by the very best (cf., e.g., [Tao11]). So one may question the wisdom of offering a new edition of the old *Basic Real Analysis*, henceforth abbreviated *BRA*. And yet, as is always the case, different people look at the same material in different ways depending on their tastes. What should or should not be included and to what extent may vary considerably, and all choices have their legitimate and logical justification.

Despite the fact that I have looked at a large number of real analysis textbooks and have benefited from all of them, I still prefer not to modify the organization of the material in *BRA*. The initial idea of a new edition came from Tom Grasso of Birkhäuser, and I want to use this opportunity to thank him for suggesting it. He pointed out that for the project to be justified, a reasonable number of changes must be made. The most substantial change in the new edition is that I rewrote Chaps. 10 and 11 on Lebesgue measure and integral entirely. In doing so, I decided to abandon F. Riesz's method used in the first edition in favor of the more traditional approach of treating Lebesgue measure *before* introducing the integral. I have come to believe that measure theory is a fundamental part of analysis and the sooner one learns it, the better.

Lebesgue measure and integral *on the real line* are now covered in Chap. 10. Chapter 11 contains additional topics, including a quick look at improper Riemann integrals, integrals depending on a parameter, the classical $\boldsymbol{L}^p$-spaces, other modes of convergence, and a final section on the *differentiation* problem. This last section contains Lebesgue's theorem on the differentiability of monotone functions (with F. Riesz's *Rising Sun Lemma* used in the proof) and his versions of the *Fundamental Theorem(s) of Calculus*. Abstract measure and integration are treated in Chap. 12, where I have included the Radon–Nikodym theorem which is used in the last section on probability.

Although the newly written chapters on Lebesgue's theory constitute the major change in this edition, all other chapters have been affected to various degrees. For example, the treatment of convex functions has been modified and (hopefully) improved. I have added a number of exercises in the text and many new problems at the end of all chapters. A large number of typographical as well as more serious errors have been corrected. I am particularly indebted to Professor Giorgio Giorgi of Università Degli Studi Di Pavia for pointing out a serious one. Of course, as always, other undetected errors may still be there and I take full responsibility for them. Needless to say, I would be grateful to careful readers for pointing them out to me (hsohrab@towson.edu).

Ideally, a book at this level should include some spectral theory, say, at least the spectral properties of compact, self-adjoint operators. Unfortunately, this would increase the size of the book beyond what I consider to be reasonable. I have decided to include some of this and similar interesting material in the end-of-chapter problems, and the interested readers may try as many of them as they want. A complete solution manual is available from the publisher for the benefit of the potential instructors. I have decided to use sequential numbering of all the items throughout. I believe that this simplifies the navigation considerably even though it may have its problems.

It is a great pleasure to thank Mitch Moulton, Birkhäuser's assistant editor, for his help and patience during the preparation of the manuscript. I am also grateful for the technical assistance I received from Birkhäuser. One of the people I completely forgot to thank in the first edition of *BRA* (shame on me!) is Loren Spice. He was 16 when he started enrolling in mathematics courses at Towson University, right when I was preparing the first draft of the textbook. He read the first five chapters very thoroughly and made a large number of suggestions and corrections. I am truly indebted to him for his valuable comments which resulted in many improvements. Also, I owe so much to the brilliant, anonymous reviewer of the first edition of *BRA* whose excellent critical comments had a great influence, even though I couldn't possibly live up to his high expectations. I hope he finds this new edition to be closer to his taste. In addition, the anonymous reviewers of this new edition have made a number of excellent comments for which I am truly grateful.

Finally, I would like to thank my wife, Shohreh, and my children, Mahsa and Zubin, whose love and support are the greatest driving force in my life.

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