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Classical Fourier Analysis

Third Edition

Springer

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To Suzanne

Preface

The great response to the publication of my book *Classical and Modern Fourier Analysis* in 2004 has been especially gratifying to me. I was delighted when Springer offered to publish the second edition in 2008 in two volumes: *Classical Fourier Analysis, 2nd Edition,* and *Modern Fourier Analysis, 2nd Edition*. I am now elated to have the opportunity to write the present third edition of these books, which Springer has also kindly offered to publish. The third edition was born from my desire to improve the exposition in several places, fix a few inaccuracies, and add some new material. I have been very fortunate to receive several hundred e-mail messages that helped me improve the proofs and locate mistakes and misprints in the previous editions.

In this edition, I maintain the same style as in the previous ones. The proofs contain details that unavoidably make the reading more cumbersome. Although it will behoove many readers to skim through the more technical aspects of the presentation and concentrate on the flow of ideas, the fact that details are present will be comforting to some. (This last sentence is based on my experience as a graduate student.) Readers familiar with the second edition will notice that the chapter on weights has been moved from the second volume to the first.

This first volume *Classical Fourier Analysis* is intended to serve as a text for a one-semester course with prerequisites of measure theory, Lebesgue integration, and complex variables. I am aware that this book contains significantly more material than can be taught in a semester course; however, I hope that this additional information will be useful to researchers. Based on my experience, the following list of sections (or parts of them) could be taught in a semester without affecting the logical coherence of the book: Sections 1.1, 1.2, 1.3, 2.1, 2.2., 2.3, 3.1, 3.2, 3.3, 4.4, 4.5, 5.1, 5.2, 5.3, 5.5, 5.6, 6.1, 6.2.

A long list of people have assisted me in the preparation of this book, but I remain solely responsible for any misprints, mistakes, and omissions contained therein. Please contact me directly (grafakosl@missouri.edu) if you have corrections or comments. Any corrections to this edition will be posted to the website

`http://math.missouri.edu/~loukas/FourierAnalysis.html` which I plan to update regularly. I have prepared solutions to all of the exercises for the present edition which will be available to instructors who teach a course out of this book.

Athens, Greece, *Loukas Grafakos*
March 2014

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I am extremely fortunate that several people have pointed out errors, misprints, and omissions in the previous editions of the books in this series. All these individuals have provided me with invaluable help that resulted in the improved exposition of the text. For these reasons, I would like to express my deep appreciation and sincere gratitude to the all of the following people.

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Finally, I would also like to thank the University of Missouri for granting me a research leave during the academic year 2013-2014. This time off enabled me to finish the third edition of this book on time. I spent my leave in Greece.

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