

# Commutative Algebra



Marco Fontana • Sophie Frisch • Sarah Glaz  
Editors

# Commutative Algebra

Recent Advances in Commutative Rings,  
Integer-Valued Polynomials, and Polynomial  
Functions

 Springer

*Editors*

Marco Fontana  
Dipartimento di Matematica  
Università degli Studi Roma Tre  
Roma, Italy

Sophie Frisch  
Mathematics Department  
Graz University of Technology  
Graz, Austria

Sarah Glaz  
Department of Mathematics  
University of Connecticut  
Storrs, CT, USA

ISBN 978-1-4939-0924-7      ISBN 978-1-4939-0925-4 (eBook)  
DOI 10.1007/978-1-4939-0925-4  
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2014943239

Mathematics Subject Classification (2010): 13-06, 13Axx, 13Bxx, 13Cxx, 13Dxx, 13Exx, 13Fxx, 13Gxx, 13Hxx, 13Jxx

© Springer Science+Business Media New York 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media ([www.springer.com](http://www.springer.com))

# Preface

This volume is mainly the outcome of a series of mini-courses and a conference on *Commutative rings, integer-valued polynomials and polynomial functions* at Technische Universität Graz, Austria, December 16–18 (mini-courses) and December 19–22 (conference), 2012. It also contains a small collection of invited articles by some of the leading experts in the area, carefully selected for the impact of their research on the major themes of the conference.

The aim of this meeting was to present recent progress in the area of commutative algebra, with primary emphasis on commutative ring theory and integer-valued polynomials along with connections to algebraic number theory, algebraic geometry and homological algebra. The wide range of topics is reflected in the table of contents of this volume. Some of the invited speakers who gave mini-courses have supplied surveys of the state of the art in newly emerging subfields.

At the conference, we had the good fortune to see that our field attracts excellent young mathematicians (who submitted good work, both individually and in collaboration with the old guard) and the not so good fortune to see that none of the young researchers have permanent jobs. May the first trend remain in full force and the second one be remedied in the near future!

Among the people and organizations who helped to make the conference and this volume of proceedings possible, our special thanks go to the departmental secretary Hermine Panzenböck and the doctoral student Roswitha Rissner, who, between the two of them, shared all the hard work of organizing, from designing the conference poster and implementing the website to applications for subsidies and the painstaking work of bookkeeping and balancing the accounts. Without their efforts, the conference would not have taken place and this volume would not have seen the light of day.

We thank the sponsors of the conference: the province of Styria, whose subsidy allowed us to sponsor the travel expenses of some graduate students and conference participants from low-income countries, and the faculty of mathematics and physics of Technische Universität Graz and the joint graduate school of natural sciences “NAWI Graz” of Technische Universität Graz and Karl-Franzens Universität Graz,

who together paid the travel expenses of all the invited speakers. Last, but not least, we thank the editorial staff of Springer, in particular Elizabeth Loew, for their cooperation, hard work and assistance with the present volume.

Rome, Italy  
Graz, Austria  
Storrs, Connecticut, USA  
December 2013

Marco Fontana  
Sophie Frisch  
Sarah Glaz

# Contents

<b>Weak Global Dimension of Prüfer-Like Rings</b> .....	1
Khalid Adarbeh and Salah-Eddine Kabbaj	
<b>Quasi-complete Semilocal Rings and Modules</b> .....	25
Daniel D. Anderson	
<b>On the Total Graph of a Ring and Its Related Graphs: A Survey</b> .....	39
Ayman Badawi	
<b>Prime Ideals in Polynomial and Power Series Rings over Noetherian Domains</b> .....	55
Ela Celikbas, Christina Eubanks-Turner, and Sylvia Wiegand	
<b>Integer-Valued Polynomials: Looking for Regular Bases (A Survey)</b> .....	83
Jean-Luc Chabert	
<b>On Boolean Subrings of Rings</b> .....	113
Ivan Chajda and Günther Eighenthaler	
<b>On a New Class of Integral Domains with the Portable Property</b> .....	119
David E. Dobbs, Gabriel Picavet, and Martine Picavet-L'Hermitte	
<b>The Probability That <math>\text{Int}_n(D)</math> Is Free</b> .....	133
Jesse Elliott	
<b>Some Closure Operations in Zariski-Riemann Spaces of Valuation Domains: A Survey</b> .....	153
Carmelo Antonio Finocchiaro, Marco Fontana, and K. Alan Loper	
<b>Ten Problems on Stability of Domains</b> .....	175
Stefania Gabelli	
<b>The Development of Non-Noetherian Grade and Its Applications</b> .....	195
Livia Hummel	

<b>Stable Homotopy Theory, Formal Group Laws, and Integer-Valued Polynomials</b> .....	213
Keith Johnson	
<b>How to Construct Huge Chains of Prime Ideals in Power Series Rings</b> ...	225
Byung Gyun Kang and Phan Thanh Toan	
<b>Localizing Global Properties to Individual Maximal Ideals</b> .....	239
Thomas G. Lucas	
<b>Prime Ideals That Satisfy Hensel’s Lemma</b> .....	255
Stephen McAdam	
<b>Finitely Stable Rings</b> .....	269
Bruce Olberding	
<b>Integral Closure of Rings of Integer-Valued Polynomials on Algebras</b> ....	293
Giulio Peruginelli and Nicholas J. Werner	
<b>On Monoids and Domains Whose Monadic Submonoids Are Krull</b> .....	307
Andreas Reinhart	
<b>Integral Closure</b> .....	331
Irena Swanson	
<b>Open Problems in Commutative Ring Theory</b> .....	353
Paul-Jean Cahen, Marco Fontana, Sophie Frisch, and Sarah Glaz	