# Part II
# Nonsmooth $\mathcal{H}_\infty$ Control

The state-space approach, which has been developed for the nonlinear $\mathcal{H}_\infty$ synthesis at the same level of generality as that in the linear case, is now extended to the non-smooth setting. Both the full information case with perfect state measurements and the incomplete information case with output disturbance-corrupted measurements are studied side by side.

Sufficient conditions for the existence of a global solution of the problem are carried out in terms of an appropriate solvability of two Hamilton–Jacobi–Isaacs partial differential inequalities that arise in the state-feedback and output-injection designs, respectively, and that may not admit continuously differentiable solutions. The present $\mathcal{L}_2$-gain analysis follows the line of reasoning where the corresponding Hamilton–Jacobi–Isaacs expressions are viewed in the sense of Clarke proximal superdifferentials and are required to be negative definite rather than semidefinite. This feature allows one to develop an $\mathcal{H}_\infty$-design procedure in the nonsmooth setting with no a priori imposed stabilizability–detectability conditions on the control system. The resulting controller is associated with specific proximal solutions of the Hamilton–Jacobi–Isaacs partial differential inequalities.

Although the design procedure results in an infinite-dimensional problem, this difficulty is circumvented by solving the problem locally. A local solution is derived by means of a certain perturbation of the DREs, which appear in solving the $\mathcal{H}_\infty$-control problem for the linearized system. By invoking the time-varying counterpart of the strict bounded real lemma, the existence of suitable solutions of the perturbed DREs is established whenever the corresponding unperturbed equations possess uniformly bounded positive semidefinite solutions. Stabilizability and detectability properties of the control system are thus ensured by the existence of the proper solutions of the unperturbed DREs, and hence the proposed synthesis procedure obviates extra work on verifying these properties that might present a formidable problem in the nonlinear case and is definitely so in the nonsmooth case.

Being developed in the general time-varying setting, the nonsmooth synthesis is then specified for periodic and autonomous systems with a focus on the periodic and, respectively, time-invariant controller design. Local output-feedback synthesis is additionally presented over sampled-data measurements. An LMI-based extension of the state-space approach to a class of nonsmooth distributed parameter systems finalizes the present development.