

Rarefied Gas Dynamics

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Translated from Russian

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Михаил Наумович Коган

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DINAMIKA RAZREZHENNOGO GAZA**

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FOREWORD

Rarefied gas dynamics, or superaerodynamics as it is sometimes called, studies phenomena taking place at an arbitrary ratio of the mean free path (time between collisions) of molecules to the characteristic dimension (time) of the phenomena. The phenomena studied can be as remote as desired from equilibrium. In the general case, an investigation of such phenomena requires consideration of the molecular structure of the gas, a kinetic description, and use of Boltzmann's equation. The range of problems of rarefied gas dynamics includes, for example, problems of flow past aircraft flying at high altitudes, motion of gases in vacuum apparatus, ultrasonic vibrations in gases, structure of shock waves, nonequilibrium flows, etc.

In the limiting case of small mean free paths we arrive at problems which can be solved within the continuum theory or, more exactly, with the use of Navier-Stokes equations. Essentially, these are problems of ordinary gas dynamics. However, according to established tradition, some of them are studied by rarefied gas dynamics. Among such problems are, for example, certain problems of viscous flows at small Reynolds numbers, flows with interaction of the boundary layer with a nonviscous flow, nonequilibrium flows with relaxation of excitation of internal degrees of freedom, flows with slip and a temperature jump near the wall, etc. Gas-dynamic methods can be used for the solution of these problems. At the same time, these problems, solvable within the continuum theory, are intimately related with kinetic theory, since, by means of kinetic theory we can derive, from an analysis of Boltzmann's equation, the Euler and Navier-Stokes equations and their analogs for relaxing media, establish their area of applicability, and supply them with correct initial and boundary conditions and transfer coefficients.

This monograph examines mainly problems requiring a kinetic description, for the solution of which the gasdynamic methods are inapplicable, and new methods, approaches, and models are needed. Prime attention is given to the Boltzmann kinetic equation, a study of its properties and methods of solution. At the same time, considerable attention is devoted to the derivation from the Boltzmann kinetic equation of equations of gasdynamics and corresponding boundary conditions (slip conditions) and to an establishment of the areas of their applicability.

The first chapter deals with basic concepts of the kinetic theory of gases. The second and third chapters are devoted to derivation of kinetic equations and general methods of their solution. Despite the fact that only gases consisting of neutral molecules are examined, some of the methods presented find use also in plasma theory, and we hope that the interpretation of the general methods given here will promote a more critical approach to their use. In the fourth chapter the use of these methods is illustrated on simple, mainly one-dimensional, problems. The fifth and sixth chapters examine limiting cases of flows at small and large Knudsen numbers.

The author's interests are, of course, reflected in the selection of the material. Some room has been given to an account of the results of works of the author and his colleagues. Certain problems which are not taken up in the book can be found by the interested reader in the literature.* The literature references do

*See, for example, S. Chapman and T. G. Cowling, "Mathematical Theory of Nonuniform Gases," Cambridge University Press, New York, 1952; J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, "Molecular Theory of Gases and Liquids," Wiley, New York, 1964; G. N. Patterson, "Molecular Flow of Gas," Wiley, New York, 1956; M. Devienne, "Flows and Heat Exchange of Rarefied Gases," [Russian translation], IL, 1962; V. P. Shidlovskii, "Introduction to Dynamics of Rarefied Gases [in Russian], Nauka Press, 1965. Numerous works on the dynamics of rarefied gases are in the proceedings of the international symposia: "Rarefied Gas Dynamics," First Symposium, Pergamon Press, New York, 1960; "Rarefied Gas Dynamics," Second Symposium, Academic Press, New York, 1961; "Rarefied Gas Dynamics," Third Symposium, Academic Press, New York, 1963; "Rarefied Gas Dynamics," Fourth Symposium, Academic Press, New York, 1966. See also: S. V. Vallander, editor, "Aerodynamics of Rarefied Gases," Leningrad State University, Vol. 1 (1963) and Vol. 2 (1965). See also: "Certain Problems in the Kinetic Theory of Gases" [Russian translation], Mir, 1965; "Interaction of Gases with Surfaces" [Russian translation], Mir, 1965.

not claim to be complete. Only works used directly in the writing of this book are cited.

Reading of the book does not require previous familiarity with the kinetic theory of gases and statistical physics. Knowledge of gasdynamics is presumed in certain places.

A course of lectures given by the author at the Moscow Physicotechnical Institute served as the basis of the book.

The author is sincerely grateful for the useful comments of Yu. P. Raizer, who carefully read the entire manuscript, A. A. Dorodnitsyn, who read its first chapters, and V. S. Galkin, who examined certain sections of the manuscript. The author is also grateful to E. M. Shakhov, who edited the book. The author would like to express special thanks to I. N. Sokolova for help in preparing the manuscript.

M. N. Kogan

NOTE TO THE AMERICAN EDITION

I am very happy to learn that my book is being translated into English, since this gives evidence of the interest of my American colleagues in the rapidly expanding field of gas dynamics. The American edition mirrors the Russian edition exactly, except that some minor errors and misprints have been corrected.

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