From Newton to Chaos

Modern Techniques for Understanding and
Coping with Chaos in N-Body Dynamical
Systems

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From Newton to Chaos

Modern Techniques for Understanding and Coping with Chaos in N-Body Dynamical Systems

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**Proceedings of a NATO Advanced Study Institute on
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in N-Body Dynamical Systems,
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in Cortina D'Ampezzo, Italy**

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Understanding and Coping with Chaos in N-Body Dynamical
Systems**
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Volume 341 — TURBULENCE: A Tentative Dictionary
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These Proceedings of the 1993 NATO Advanced Study
Institute held in Cortina d'Ampezzo, Italy
between July 25 and August 6
are dedicated to the
memory of

Henri Poincaré (1854 - 1912)

distinguished French mathematician and theoretical astronomer,
pioneer in the definition and study of chaos.

PREFACE

The reader will find in this volume the Proceedings of the NATO Advanced Study Institute held in Cortina d'Ampezzo, Italy, between July 25 and August 6, 1993, under the title From Newton to Chaos: Modern Techniques for Understanding and Coping With Chaos in N-Body Dynamical Systems.

This institute was the latest in a series of meetings held every three years from 1972 to 1990 in dynamical astronomy, theoretical mechanics and celestial mechanics. The proceedings from these institutes have been well-received in the international community of research workers in these disciplines. The present institute was well attended with 15 series of lectures being given by invited speakers: in addition some 40 presentations were made by the other participants. The majority of these contributions are included in these proceedings.

The all-pervading influence of chaos in dynamical systems (of even a few variables) has now been universally recognised by researchers, a recognition forced on us by our ability, using powerful computer hardware and software, to tackle dynamical problems that until twenty-five years ago were intractable. Doubtless it was felt by many that these new techniques provided a break-through in celestial mechanics and its related disciplines. And so they were. It was wonderful to be able to integrate the orbits of a star cluster and obtain some of the properties of the cluster's evolution; or to study the orbits of the outer planets over periods of 10^8 years or the inner planets for some 10^7 years; or to follow the pinball progress of a comet through the solar system.

Nevertheless the part played by chaos in all these examples - and many more - is clear. The existence of the predictability horizon of the given problem, a time limit beyond which no confidence can be placed in the calculations, has been recognised. Within such predictability horizons our armoury of analytical and computer techniques is able to produce results accurate enough for all practical purposes. Nevertheless there remain many dynamical problems that still escape our understanding because of their essentially chaotic nature.

It is a measure of the genius of the great French mathematician and theoretical astronomer Henri Poincaré, that almost a century ago, he grasped clearly the concept of chaos and its prevailing influence not only in dynamical systems but in human life. Some of his contributions to our understanding of the part chaos plays will be found in this volume. Lesser mortals such as ourselves can only marvel at his brilliant farsightedness and acknowledge it in some small way by respectfully dedicating this volume to his memory. In it, under five sections, namely: Planetary Theory; Dynamics of Asteroids, Comets and Meteors; Dynamics of Natural and Artificial Satellites; The Three Body Problem; and Selected Topics in Dynamics, will be found the contributions of this NATO

ASI. There is no section devoted exclusively to chaos: there cannot be for chaos is universal.

Many more researchers applied to participate in this NATO ASI than could be accommodated. Nevertheless, the Organising Committee was particularly happy to be able, with NATO's help, to invite some of its colleagues from a number of eastern European countries. The warm international friendship demonstrated by the participants during the duration of the NATO ASI was a most valuable feature of the meeting.

The Organising Committee (Professor A. E. Roy (Director), Professor V. Szebehely (Associate Director), Dr P. J. Message, Professor A. Milani and Dr B. A. Steves) would like to take this opportunity to thank all those whose contributions made the ASI a success and such a very happy occasion. In particular the Committee would like to thank the staff of the Antonelli Institute where the ASI was held. As always they were unstintingly helpful and friendly.

The Committee is also sincerely grateful to the Scientific Affairs Division for their guidance, counsel and support.

The editors of this volume would like to take the opportunity of thanking Andrew Conway, Sarah Matthew, Elaine Rowan and David Ramsay of the Department of Physics and Astronomy of Glasgow University for their valuable help in preparing some of the manuscripts. The editors would also like to thank all those who came to Cortina and who supplied papers for this volume. Their cooperation and friendship are very much appreciated.

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