Digital Filtering in One and Two Dimensions

Design and Applications

Digital Filtering in One and Two Dimensions

Design and Applications

Robert King

*Imperial College
London, England*

Majid Ahmadi

*University of Windsor
Windsor, Ontario, Canada*

Raouf Gorgui-Naguib

*University of Newcastle upon Tyne
Newcastle upon Tyne, England*

Alan Kwabwe

*Imperial College
and Bankers Trust Company
London, England*

and

Mahmood Azimi-Sadjadi

*Colorado State University
Fort Collins, Colorado*

Springer Science+Business Media, LLC

Library of Congress Cataloging in Publication Data

Digital filtering in one and two dimensions: design and applications / Robert King
. . . [et al.].

p. cm.

Includes bibliographies and index.

ISBN 978-1-4899-0920-6

1. Digital filters (Mathematics) 2. Signal processing—Digital techniques. 3. Transformations (Mathematics) I. King, Robert Ashford.

QA297.D54 1989

88-30268

621.38'043—dc19

CIP

ISBN 978-1-4899-0920-6

ISBN 978-1-4899-0918-3 (eBook)

DOI 10.1007/978-1-4899-0918-3

© 1989 Springer Science+Business Media New York

Originally published by Plenum Press, New York in 1989

Softcover reprint of the hardcover 1st edition 1989

All rights reserved

No part of this book may be reproduced, stored in a retrieval system, or transmitted
in any form or by any means, electronic, mechanical, photocopying, microfilming,
recording, or otherwise, without written permission from the Publisher

**To
Gillian
Jocelyne
May
Yetunde
Sheida**

Preface

This book has been conceived to extend the generally published work on one- and two-dimensional digital filters in order to include some of the more recently developed ideas. It is intended to supplement and build on the classical books which cover the fundamental concepts of the topic. As a consequence of this, the basic theory is stated in a compact manner and is not developed thoroughly, as this would result in considerable duplication of existing books. The main theme of the book has been to provide a comprehensive background to the methods available for the realization of both recursive and nonrecursive digital filters, and to give an insight into some of the more recent implementation procedures.

The book is planned to cover one- and two-dimensional systems in parallel, showing the techniques which are applicable in both areas, and also the limitations and constraints necessary when a one-dimensional technique is extended to systems of higher dimensionality. The theme of the book commences with several chapters on the design of filter transfer functions to meet given specifications. This is followed by a discussion of methods of implementing these in a practical system and the limitations imposed as a result of noise and finite word length. Finally, a discussion of some applications is included.

The first chapter includes a brief review of the theory which is necessary in the remainder of the book; it covers some of the fundamental topics of convolution, Z transformation, and Fourier transforms without any attempt at mathematical rigor or completeness. These topics are already well covered in a number of existing texts.

This is followed by three chapters which provide a detailed study of the techniques available for the design of both nonrecursive and recursive filter transfer functions. In the design of nonrecursive digital filters, the use of windows in both one and two dimensions is considered. This is followed by a discussion of the frequency sampling technique and the use of iterative

methods for the design of nonrecursive filters. Finally, the use of frequency transformations in two dimensions is studied.

A chapter follows which briefly reviews the conditions for stability of one-dimensional systems and the techniques available for the stabilization of inherently unstable filters. The extension to two dimensions is taken to considerable length in order to show the greatly increased complexity of the problem and the various attempts which have been made to solve the two-dimensional stabilization problem for a range of filter functions. A description of several different methods of generating stable polynomials is offered next.

This is followed by a chapter on recursive filters in both one and two dimensions, in which the objective is to design a filter to fit a given specification with no danger of incurring problems of instability. Most of the techniques discussed here are based in the frequency domain; however, a short outline of some of the more profitable space-domain two-dimensional filter design methods is included.

The next chapter gives an overall view of the types of errors which occur in digital filters as a result of the discrete nature of the process; the finite word length representing the values of data and filter coefficients; and the errors which occur as a result of overflow in the multiplication process. The limit cycle oscillations which may be generated as a result of quantization in the system are also considered.

The following three chapters cover some methods of implementing the designed transfer functions in terms of the basic multipliers, adders, and delay elements. The first technique considered is that whereby an analogue filter is designed as a ladder network and then transformed by means of a set of linear equations into a form analogous to wave equations. These may now be digitized, giving a direct implementation as a digital system. This retains the advantage of the good sensitivity of analogue structures.

The second approach is via the state-space representation of filter functions. In this form, it is relatively simple to obtain a general transformation matrix which may be chosen to realize a large body of implementations of the transfer functions; these different realizations may have the advantage of low noise, consistent with limited dynamic range, minimum number of elements, or other properties.

A third approach to the implementation of systems processing large arrays of data is that of breaking the data into subunits and processing these blocks individually, aggregating them at the output to give the complete resultant image. Several methods of performing this are considered.

The next chapter digresses to consider the implementation of a filter function by transforming the data into some alternative domain, and a number of Fourier-like methods, mainly in number systems defined in a

finite field, are considered. Some of the newer systems, such as polynomial transforms and p -adic transforms, are studied.

There follows a chapter on image modeling; this is concerned with the introduction of two-dimensional causal, semicausal, and noncausal autoregressive moving-average models and the related issues of parameter estimation, spectral factorization, stability, and convergence. The applications are in the restoration of satellite and radar images.

The final two chapters are devoted to a consideration of the general philosophy of the application of digital filters to practical problems. The first of these two chapters deals with the general methodology of applying digital filters to real problems, and the types of filter functions needed in many cases. The second chapter takes four case studies and shows how filtering may be used to improve or enhance those properties of an image which are of interest in a given application.

We should like to record our thanks and gratitude to a number of our colleagues, both past and present, who have either knowingly or unwittingly contributed to this book. In particular, we should mention Drs. G. A. Jullien, V. Ramachandran, and S. Udpa, who read and commented on parts of the manuscript and suggested modifications and improvements. The assistance of Ahmet Kayran, Nahy Nassar, Amar Ali, Desmond McLernon, Nasser Nasrabadi, Behrouz Nowrouzian, Karim Henein, and H. Lee is also gratefully acknowledged.

Finally, our gratitude goes to Christine Lane and Shirley Ovellette, who produced an excellent typescript from our less than perfect manuscripts.

Contents

<i>Chapter 1. Discrete Time/Space Linear Filtering</i>	
1.1. Introduction	1
1.2. Digital Systems	2
1.2.1. Causality	3
1.2.2. Recursion Directions of Finite Area Array	4
1.3. Recursion Equations	5
1.3.1. Convolution Equations	6
1.3.2. Impulse Response	7
1.4. Fourier Transform	7
1.4.1. Periodicity	9
1.4.2. Symmetry	9
1.4.3. Translation	10
1.4.4. Rotation	10
1.4.5. Circularly Symmetric Arrays	10
1.4.6. Relation between Discrete and Continuous Fourier Transforms	11
1.5. Z Transforms	11
1.5.1. Z Transform of Delayed Function	12
1.5.2. Z Transform of Convolution	13
1.5.3. Inverse Z Transform	13
1.5.4. Relation between Z Transform and Discrete Fourier Transform	15
1.6. Solution of Recursion Equations	15
1.6.1. Z -Transform Solution—Transfer Function	15
1.6.2. Frequency Response	17
1.7. Fast Fourier Transform	18
1.7.1. Decimation-in-Time Algorithm	19
1.7.2. Decimation-in-Frequency Algorithm	22
1.7.3. Computation of Inverse Discrete Fourier Transform	23

1.7.4. Data Shuffling	24
1.7.5. Number of Multiplications for One-Dimensional Fast-Fourier-Transform Algorithm	25
1.7.6. Two-Dimensional Fast Fourier Transform	26
1.7.7. Number of Multiplications for Two-Dimensional Fast Fourier Transform	26
1.8. Convolution	27
1.8.1. Matrix Representation of Convolution	27
1.8.2. Computation of Convolution	28
References	30

Chapter 2. Nonrecursive Filters

Part One. One-Dimensional Filters

2.1. Introduction	31
2.2. Design of Finite-Impulse-Response Filters using the Fourier-Series Method	34
2.3. Use of Window Functions	37
2.3.1. Hann and Hamming Windows	38
2.3.2. Blackman Window	38
2.3.3. Kaiser Window	40
2.4. Design of Nonrecursive One-Dimensional Filters using a Modified Kaiser Window	44
2.5. Frequency Sampling Method	47
2.6. Equiripple Design of a Nonrecursive Filter	48
2.6.1. Problem Formulation	49
2.6.2. Remez Exchange Algorithm	51
2.6.3. Implementation of Algorithm	52

Part Two. Two-Dimensional Filters

2.7. Introduction	54
2.8. Design of Two-Dimensional Nonrecursive Filters using Windows	55
2.8.1. Two-Dimensional Window Function	56
2.9. Optimal Design of Two-Dimensional Nonrecursive Filters using Kaiser Windows	57
2.10. McClellan Transformation	59
2.10.1. The Contour Approximation Problem	62
References	63

Chapter 3. Stability and Stabilization Techniques

Part One. One-Dimensional System

3.1. Introduction	65
3.2. Testing the Stability of One-Dimensional Recursive Filters	67

Contents	xiii
3.3. Generation of One-Dimensional Stable Polynomials	69
3.4. Stabilization of One-Dimensional Recursive Digital Filters	70
3.4.1. Pole-Zero Cancellation Technique	70
3.4.2. Stabilization via a Discrete Hilbert Transform	71
Part Two. Two-Dimensional System	
3.5. Introduction	77
3.6. Ansell Stability Theory	80
3.7. The Anderson and Jury Stability Test	82
3.7.1. Denominator Examination with z_1 Limited	82
3.7.2. Denominator Examination with both z_1 and z_2 Limited	84
3.8. The Maria and Fahmy Method for Testing Stability	87
3.9. Stabilization of Two-Dimensional Recursive Digital Filters	89
3.10. Double Planar Least-Squares Inverse Method	90
3.10.1. Definitions	91
3.11. Stabilization via Two-Dimensional Discrete Hilbert Transforms	92
3.12. Stabilization of Two-Dimensional Zero-Phase Filters	95
3.13. Generation of Two-Dimensional Digital Functions without Nonessential Singularities of the Second Kind	98
3.13.1. Value of a Function at Infinity	99
3.13.2. Singularities	99
3.13.3. Two-Variable Hurwitz Polynomials	100
3.13.4. Generation of a Very Strict Hurwitz Polynomial using Properties of the Derivatives of Even or Odd Parts of Hurwitz Polynomials	102
3.13.5. Generation of Two-Variable Very Strict Hurwitz Poly- nomials using Matrix Theory	108
References	116
 <i>Chapter 4. Recursive Filters</i>	
Part One. One-Dimensional Filters	
4.1. Introduction	119
4.2. Techniques for Discretization of Analogue Transfer Functions	121
4.2.1. Mapping of Differentials	121
4.2.2. Impulse Invariant Transformation	123
4.2.3. Bilinear Transformation	125
4.2.4. Matched z Transform	128
4.3. Direct Design of Recursive Digital Filters	129
4.3.1. Linear Programming Approach	129
4.3.2. Modified Linear Programming Approach	131
4.3.3. Nonlinear Programming Approach	138
4.4. Analogue Filter Approximations	144
4.4.1. The Butterworth Approximation	145
4.4.2. Analogue-to-Analogue Transformation	148

4.4.3. Design of Low-Pass Butterworth Filters	150
4.4.4. Design of Band-Pass Butterworth Filters	152
4.5. Chebyshev Filters	154
4.6. Digital-to-Digital Transformation	159
Part Two. Two-Dimensional Filters	
4.7. Introduction	160
4.8. Separable Product Filters	162
4.9. Separable Denominator, Nonseparable Numerator Filters	164
4.10. Nonseparable Numerator and Denominator Filters	166
4.11. Transformation Techniques	167
4.11.1. Rotated Filters	167
4.11.2. Transformation Technique using a Two-Variable Reactance Function	171
4.11.3. Complex Transformations and Their Applications in Two-Dimensional Fan-Filter Design	178
4.12. Iterative Approach to the Design of Two-Dimensional Filters	185
4.12.1. Linear Programming Approach	185
4.12.2. Nonlinear Programming Approach	190
4.13. Spatial Design Technique	191
4.13.1. The Shanks, Treitel, and Justice Method	191
4.13.2. The Bordner Technique	194
References	195
 <i>Chapter 5. Quantization and Roundoff Errors</i>	
5.1. Sources of Errors in Digital Filters	197
5.2. Input Quantization Effects	198
5.3. Types of Arithmetic	200
5.4. Roundoff Noise	203
5.5. Effect of Filter Realization on the Noise Model	207
5.6. Limit-Cycle Oscillations in Digital Filters	209
5.6.1. Reverse Limit Cycle	210
5.6.2. Phase-Plane Symmetry	210
5.6.3. Limit Cycle Properties	211
5.7. Digital Filtering using the Logarithmic Number System	213
References	216
 <i>Chapter 6. Direct Realization and Implementation Using Linear Transformations</i>	
6.1. Introduction	219
6.2. Classical Canonic Implementation	219
6.2.1. Direct Canonic Form	220
6.2.2. Parallel Realization	220
6.2.3. Cascade Realization	222

6.3. Implementation Based on Ladder Networks	222
6.3.1. Flow Graph of Analogue Prototype Filter	223
6.4. Implementation of One-Dimensional Digital Filters	231
6.4.1. Transformation Set with Unity Cycle Length	231
6.4.2. Transformation Set with Cycle Length of Two	239
6.4.3. Full Length Cycle Transformations	241
6.5. Two-Dimensional Implementation	243
6.5.1. Realization using Unity Cycle Length	244
References	247

Chapter 7. State-Space Description

7.1. One-Dimensional State-Space Models	250
7.1.1. Realization of State-Space Model	254
7.2. One-Dimensional Structure Transformation	254
7.2.1. Effect of Translation	254
7.2.2. Effect of Rotation and Scaling	255
7.2.3. Fully Decoupled States	255
7.3. Solution of One-Dimensional State Equations	258
7.3.1. Time-Domain Solution	258
7.3.2. Z-Domain Solution	259
7.3.3. State Transition Matrix	260
7.3.4. Stability of One-Dimensional Systems	262
7.3.5. Controllability and Observability in One-Dimensional Systems	262
7.4. Representation of One-Dimensional State Operations	264
7.5. Two-Dimensional State-Space Models	265
7.6. Two-Dimensional Structure Transformation	269
7.7. Realization of State-Space Equations	269
7.8. Solution of Two-Dimensional State-Space Equations	272
7.8.1. State Transition Matrix	272
7.8.2. Spatial-Domain Solution	273
7.8.3. Z-Domain Solution	274
7.8.4. The Characteristic Polynomial and the Cayley-Hamilton Theorem for Two-Dimensional Systems	276
7.8.5. Stability of Two-Dimensional Systems	277
7.8.6. Controllability and Observability in Two-Dimensional Systems	279
7.9. Roundoff Error in State-Space Digital Filters	285
7.9.1. Roundoff Errors in One-Dimensional Systems	285
7.9.2. Dynamic-Range Constraint in One-Dimensional Systems	287
7.9.3. Output Noise Power in One-Dimensional Systems	289
7.9.4. Solution of the Minimization Problem in the One-Dimensional Case	290

7.9.5. Roundoff Errors in Two-Dimensional Systems	291
7.9.6. Dynamic-Range Constraint in Two-Dimensional Systems	292
7.9.7. Output Noise Power in Two-Dimensional Systems	293
7.9.8. Solution of the Minimization Problem in the Two-Dimensional Case	294
7.9.9. Example of Minimization of the Noise Figure for a One-Dimensional Filter	295
References	296

Chapter 8. Block Implementation of Digital Filters

8.1. Implementation of Finite-Impulse-Response Digital Filters	300
8.1.1. Select-Save Method	302
8.1.2. Overlap-Add Method	303
8.1.3. Minimization of Computation Time	303
8.2. Implementation of Infinite-Impulse-Response Digital Filters	304
8.2.1. One-Dimensional Block Processing	305
8.2.2. Computational Methods for One-Dimensional Block Recursive Implementation	310
8.2.3. Two-Dimensional Block Processing	312
8.2.4. Computational Methods for Two-Dimensional Block Recursive Implementation	317
8.3. Block State-Space Implementation	321
8.3.1. One-Dimensional Block State-Space Implementation	321
8.3.2. Stability, Observability, and Controllability of Block State-Space Systems	323
8.3.3. Two-Dimensional Block State-Space Implementation	325
8.3.4. Stability of the Two-Dimensional Block State-Space System	329
8.4. Application of Block-Processing Techniques	330
References	331

Chapter 9. Number Theoretic Transformation Techniques

9.1. Orthogonal Transforms and Elements of Modular Arithmetic	333
9.1.1. Divisibility of Integers	334
9.1.2. Congruences and Residues	334
9.1.3. Groups and Abelian Groups	335
9.1.4. Rings and Fields	335
9.1.5. Modular Arithmetic	336
9.1.6. The Chinese Remainder Theorem	336
9.1.7. Euler's Totient Function, Euler's Theorem, and Fermat's Little Theorem	337
9.1.8. Quadratic Residues and the Legendre Symbol	338

9.1.9. Extension Fields of Integer Fields, I_p	338
9.1.10. Residue Polynomials	338
9.2. Number Theoretic Transforms	339
9.2.1. Number Theoretic Transform over a Field, I_p	339
9.2.2. Number Theoretic Transform over a Ring, Z_M	340
9.2.3. Mersenne and Fermat Number Transforms	341
9.2.4. Complex Number Theoretic Transform	343
9.2.5. Quadratic Number Theoretic Transform	344
9.2.6. Number Theoretic Transforms in Galois Rings $GF(q^2)$	345
9.3. Polynomial Transform	346
9.4. p -Adic Transform	349
9.4.1. The p -adic Field, Q_p , and the Segmented p -adic Field, $\hat{Q}_p$	349
9.4.2. The General p -adic Transform	351
9.4.3. The Existence and Derivation of $H(p, r, \gamma)$ in $\hat{Q}_p$	352
9.4.4. Mersenne and Fermat p -adic Transforms	353
9.4.5. Complex p -adic Transform	355
9.4.6. Complex Convolution Via a Fermat p -adic Transform	356
9.4.7. Quadratic p -adic Transform in Extension Fields of Q_p with p a Mersenne Prime	357
9.4.8. Quadratic p -adic Transform in Extension Fields of Q_p with p a Fermat Prime	360
9.4.9. p -adic Transform Defined in a g -adic Ring Over a Direct Sum of Several p -adic Fields	360
9.5. The Use of Transforms Based on Number Theoretic Algorithms	362
References	363

Chapter 10. Image Modeling

10.1. One-Dimensional Autoregressive, Moving Average, and Auto- regressive Moving Average Models	368
10.1.1. Causal Representation	368
10.1.2. Noncausal Representation	374
10.2. Two-Dimensional Linear Prediction Models	379
10.2.1. Causal Representation	379
10.2.2. Semicausal Representation	386
10.2.3. Noncausal Representation	391
10.3. Vector Models	395
10.3.1. Vector Causal Models	396
10.3.2. Vector Semicausal Models	397
References	400

Chapter 11. Applications

11.1. Introduction	401
11.2. Digital Image Processing—Models	403

11.3. Image Enhancement	405
11.3.1. Enhancement to Reduce the Effect of Noise	406
11.3.2. Edge Enhancement	410
11.4. Image Restoration	411
11.4.1. Linear Nonrecursive Filtering	412
11.4.2. Linear Recursive Restoration Filters	417
11.4.3. Nonlinear and Space-Variant Restoration Filtering	421
11.5. Image Reconstruction	423
11.5.1. Reconstruction from Complete Information	423
11.5.2. Reconstruction from Incomplete Information	423
References	425
 <i>Chapter 12. Case Studies</i>	
12.1. Introduction	427
12.2. Case Study 1: X-ray Image Analysis	427
12.2.1. Background	428
12.2.2. X-ray Image Filtering Examples	430
12.2.3. Summary	432
12.3. Case Study 2: Intelligent Image-Processing Systems for Quality Control of Manufactured Parts	432
12.3.1. Background	433
12.3.2. Filter	434
12.3.3. Car Piston Head Casting	436
12.3.4. Differential Gear Wheels	437
12.3.5. Strain Gauges	438
12.3.6. Summary	439
12.4. Case Study 3: Restoration of Linearly Degraded Images with Additive Noise	439
12.4.1. Background	439
12.4.2. Restoration Example	440
12.4.3. Summary	444
12.5. Case Study 4: Image Reconstruction from Incomplete Information	445
12.5.1. Background	445
12.5.2. Reconstruction Examples	448
12.5.3. Results	455
12.5.4. Summary	455
References	458
 Index	 459

Explanatory List of Abbreviations

AR	autoregressive
ARMA	autoregressive moving average
BHP	broad sense Hurwitz polynomial
BIBO	bounded-input, bounded-output
CNTT	complex number theoretic transform
CRT	Chinese Remainder Theorem
CT	computerized tomography
DFT	discrete Fourier transform
DR	dynamic range
FFT	fast Fourier transform
FIR	finite impulse response
FNT	Fermat number transform
HP	Hurwitz polynomial
HVS	human visual system
IDFT	inverse discrete Fourier transform
IIR	infinite impulse response
LHP	left-half plane
LP	low pass
MA	moving average
MIMO	multi-input, multi-output
MNT	Mersenne number transform
MVR	minimum variance representation
NHP	narrow sense Hurwitz polynomial
NSHP	nonsymmetrical half-plane
NTT	number theoretic transform
PAT	p -adic transform
PLSI	planar least-squares inverse
PPS	phase-plane symmetry
PSF	point spread function

PT	polynomial transform
RLC	reverse limit cycle
SDF	spatial density function
SHP	strict Hurwitz polynomial
SISO	single-input, single-output
	summable-input, summable-output (Chapter 3)
VSHP	very strict Hurwitz polynomial