

MATLAB Numerical Calculations



César Pérez López

Apress®

MATLAB Numerical Calculations

Copyright © 2014 by César Pérez López

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

ISBN-13 (pbk): 978-1-4842-0347-7

ISBN-13 (electronic): 978-1-4842-0346-0

Trademarked names, logos, and images may appear in this book. Rather than use a trademark symbol with every occurrence of a trademarked name, logo, or image we use the names, logos, and images only in an editorial fashion and to the benefit of the trademark owner, with no intention of infringement of the trademark.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Managing Director: Welmoed Spahr

Lead Editor: Jeffrey Pepper

Editorial Board: Steve Anglin, Mark Beckner, Gary Cornell, Louise Corrigan, Jim DeWolf, Jonathan Gennick,

Robert Hutchinson, Michelle Lowman, James Markham, Matthew Moodie, Jeff Olson, Jeffrey Pepper,

Douglas Pundick, Ben Renow-Clarke, Gwenan Spearing, Steve Weiss

Coordinating Editor: Melissa Maldonado

Copy Editor: Barnaby Sheppard

Compositor: SPi Global

Indexer: SPi Global

Artist: SPi Global

Cover Designer: Anna Ishchenko

Distributed to the book trade worldwide by Springer Science+Business Media New York, 233 Spring Street, 6th Floor, New York, NY 10013. Phone 1-800-SPRINGER, fax (201) 348-4505, e-mail orders-ny@springer-sbm.com, or visit www.springeronline.com. Apress Media, LLC is a California LLC and the sole member (owner) is Springer Science + Business Media Finance Inc (SSBM Finance Inc). SSBM Finance Inc is a Delaware corporation.

For information on translations, please e-mail rights@apress.com, or visit www.apress.com.

Apress and friends of ED books may be purchased in bulk for academic, corporate, or promotional use. eBook versions and licenses are also available for most titles. For more information, reference our Special Bulk Sales–eBook Licensing web page at www.apress.com/bulk-sales.

Any source code or other supplementary material referenced by the author in this text is available to readers at www.apress.com. For detailed information about how to locate your book's source code, go to www.apress.com/source-code/.

Contents at a Glance

About the Author	xi
Introduction	xv
■ Chapter 1: Introduction to MATLAB	1
■ Chapter 2: Integers, Divisibility and Number Systems	11
■ Chapter 3: Real and Complex Numbers	43
■ Chapter 4: Numerical Variables, Vectors and Matrices	95
■ Chapter 5: Vectors and Matrices	139
■ Chapter 6: Functions	179
■ Chapter 7: Programming and Numerical Analysis	211
■ Chapter 8: Numerical Algorithms: Equations, Derivatives, Integrals and Differential Equations	279

Contents

About the Author	xi
Introduction	xv
■ Chapter 1: Introduction to MATLAB	1
1.1 Numerical Calculations with MATLAB	1
1.2 Symbolic Calculations with MATLAB	3
1.3 MATLAB and Maple	5
1.4 General Notation. The Command Window	5
1.5 MATLAB and Programming.....	8
1.6 Translating C, FORTRAN and TEX expressions.....	9
■ Chapter 2: Integers, Divisibility and Number Systems	11
2.1 Arithmetic Operations in MATLAB.....	11
2.2 Integers	16
2.3 Divisibility.....	17
2.4 Modular Arithmetic.....	28
2.5 Divisibility in $Z[\sqrt{n}]$	32
2.6 Diophantine Equations	35
2.7 Number Systems.....	36
■ Chapter 3: Real and Complex Numbers	43
3.1 Rational Numbers	43
3.2 Continued Fractions	51
3.3 Irrational Numbers	55
3.4 Algebraic Numbers.....	62

3.5 Real Numbers.....	63
3.6 Common Functions with Real Arguments	63
3.7 Complex Numbers.....	66
3.8 Common Functions with Complex Arguments	66
3.9 Divisibility in the Complex Field. The Ring of Gaussian Integers	79
3.10 Approximation and Precision.....	85
3.11 Types of Numbers and Expressions.....	89
3.12 Random Numbers.....	92
■ Chapter 4: Numerical Variables, Vectors and Matrices	95
4.1 Variables.....	95
4.2 Variables and Special Constants	98
4.3 Symbolic and Numeric Variables.....	100
4.4 Vector Variables.....	105
4.5 Matrix Variables.....	110
4.6 Character Variables	119
4.7 Operators.....	122
4.7.1 Arithmetic Operators.....	122
4.7.2 Relational Operators	125
4.7.3 Logical Operators	126
4.8 Logic Functions	126
4.9 Elementary Functions that Support Complex Matrix Arguments.....	129
4.10 Elementary Functions that Support Complex Vector Arguments.....	133
4.11 Vector Functions of Several Variables	136
4.12 Functions of One Variable.....	137
■ Chapter 5: Vectors and Matrices	139
5.1 Vectors and Matrices.....	139
5.2 Operations with Numeric Matrices.....	140
5.3 Eigenvalues and Eigenvectors.....	150
5.4 Matrix Decomposition	156

5.5 Similar Matrices and Diagonalization..... 169

5.6 Sparse Matrices 171

5.7 Special Matrices..... 173

■ **Chapter 6: Functions 179**

6.1 Custom Defined Functions 179

6.2 Functions and M-files..... 179

6.3 Functions and Flow Control. Loops 183

6.4 The FOR loop 183

6.5 The WHILE loop..... 185

6.6 IF ELSEIF ELSE END LOOP 186

6.7 Recursive Functions 187

6.8 Conditional Functions..... 190

6.9 Defining Functions Directly. Evaluating Functions 193

6.10 Functions of One Variable..... 193

6.11 Functions of Several Variables 194

6.12 Piecewise Functions 198

6.13 Functional Operations 204

■ **Chapter 7: Programming and Numerical Analysis 211**

7.1 MATLAB and Programming..... 211

7.2 The Text Editor 211

7.3 Scripts 214

7.4 Functions and M-files. Eval and feval 217

 7.4.1 A Simple Function Definition 122

7.5 Local and Global Variables 220

7.6 Data Types 222

7.7 Flow Control: FOR, WHILE and IF ELSEIF Loops..... 223

7.8 FOR Loops 223

7.9 WHILE Loops..... 224

7.10 IF ELSEIF ELSE END Loops	225
7.11 SWITCH and CASE	227
7.12 CONTINUE.....	228
7.13 BREAK	229
7.14 TRY ... CATCH.....	230
7.15 RETURN	230
7.16 Subfunctions	231
7.17 Commands in M-files	232
7.18 Functions relating to Arrays of Cells.....	233
7.19 Functions of Multidimensional Arrays.....	236
7.20 Numerical Analysis Methods in MATLAB.....	240
7.21 Zeros of Functions and Optimization.....	240
7.22 Numerical Integration.....	243
7.23 Numerical Differentiation	244
7.24 Approximate Solutions of Differential Equations.....	246
7.25 Ordinary Differential Equations with Initial Values	246
7.26 Ordinary Differential Equations with Boundary Conditions	249
7.27 Partial Differential Equations	252
■ Chapter 8: Numerical Algorithms: Equations, Derivatives, Integrals and Differential Equations	279
8.1 Solving Non-Linear Equations	279
8.1.1 The fixed Point Method for Solving $x = g(x)$	279
8.1.2 Newton's Method for Solving the Equation $f(x) = 0$	282
8.1.3 Schröder's Method for Solving the Equation $f(x)=0$	284
8.2 Systems of Non-Linear Equations	284
8.2.1 The Seidel Method.....	284
8.2.2 The Newton-Raphson Method	285

8.3 Interpolation Methods	288
8.3.1 Lagrange Polynomial Interpolation	288
8.3.2 Newton Polynomial Interpolation.....	290
8.4 Numerical Derivation Methods	291
8.4.1 Numerical Derivation via Limits.....	291
8.4.2 Richardson’s Extrapolation Method	294
8.4.3 Derivation Using Interpolation (n + 1 Nodes).....	295
8.5 Numerical Integration Methods	297
8.5.1 The Trapezium Method	297
8.5.2 Simpson’s Method	300
8.6 Ordinary Differential Equations	302
8.6.1 Euler’s Method.....	302
8.6.2 Heun’s Method.....	303
8.6.3 The Taylor Series Method	304

About the Author

César Pérez López is a Professor at the Department of Statistics and Operations Research at the University of Madrid. César is also a Mathematician and Economist at the National Statistics Institute (INE) in Madrid, a body which belongs to the Superior Systems and Information Technology Department of the Spanish Government. César also currently works at the Institute for Fiscal Studies in Madrid.

Also Available

- *MATLAB Programming for Numerical Analysis*, 978-1-4842-0296-8
- *MATLAB Control Systems Engineering*, 978-1-4842-0290-6
- *MATLAB Differential Equations*, 978-1-4842-0311-8
- *MATLAB Linear Algebra*, 978-1-4842-0323-1
- *MATLAB Differential and Integral Calculus*, 978-1-4842-0305-7
- *MATLAB Optimization Techniques*, 978-1-4842-0293-7
- *MATLAB Symbolic Algebra and Calculus Tools*, 978-1-4842-0344-6

Introduction

MATLAB is the tool of the modern day mathematician or engineer. The incredible deep functionality of MATLAB makes what would have taken hours 40 years ago, often less than a minute with MATLAB built in functions. MATLAB enables you to explore multiple approaches and reach a solution faster often more accurately than with other tools or traditional programming languages, such as C/C++ or Java. More importantly, it has changed the way we learn and made getting solutions immensely simpler. So, you can now focus on the application, instead of the math. And with that easily available power, you can explore and find more and more functions, test hypotheses and become a significantly more powerful worker.

This book is designed for use, in part, as a tool to enable you to use MATLAB as a scientific/business calculator so that you can get numerical solutions to problems involving a wide array of mathematics using MATLAB. But it is broader in scope than that. The book can be used as an independent resource for MATLAB. A background in the necessary mathematics is assumed, so this book shows you how to interpret your problems to get MATLAB to do what you want it to do. Just look up the function you want in the book and you are ready to use it in MATLAB or use the book to learn about the enormous range of options that MATLAB offers. The book is topical, picking examples to show not only general methods in using MATLAB, but specifics to use MATLAB for advanced mathematical computations while giving a glimpse at their application.

MATLAB Numerical Calculations focuses on MATLAB capabilities to give you numerical solutions to problems you are likely to encounter in your professional or scholastic life. It introduces you to the MATLAB language with practical hands-on instructions and results, allowing you to quickly achieve your goals. Starting with a look at basic MATLAB functionality with integers, rational numbers and real and complex numbers, and MATLAB's relationship with Maple, you will learn how to solve equations in MATLAB, and how to simplify the results. You will see how MATLAB incorporates vector, matrix and character variables, and functions thereof. MATLAB is a powerful tool used to defined, manipulate and simplify complex algebraic expressions. With MATLAB you can also work with ease in matrix algebra, making use of commands which allow you to find eigenvalues, eigenvectors, determinants, norms and various matrix decompositions, among many other features. Lastly, you will see how you can write scripts and use MATLAB to explore numerical analysis, finding approximations of integrals, derivatives and numerical solutions of differential equations.