Complex Analysis and Geometry

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Daniel Gorenstein
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Edited by Vincenzo Ancona and Alessandro Silva

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Preface

When we studied complex variables in the late 1960s, modern geometry on the complex field and complex function theory were identified in teaching and research as several complex variables. A beginner in the field at that time would have the experience of jumping from the sheaf-theoretical methods employed in the theory of analytic spaces to the P.D.E. methods of the $\bar{\partial}$-problem, with the clear understanding that the phenomena lying behind such different methods and problems were the same. A few years later, new important discoveries made clear that complex differential geometry was also in the same company.

Looking at the historical development of the subject in the first half of the twentieth century shows this was not astonishing. The origin of the theory of functions of several complex variables was tardier than the familiar theory of analytic functions of one complex variable. The first comprehensive textbook by Behnke and Thullen, in the 1930s, expounded the foundations of the general theory as set up by Weierstrass, Cousin, Hartogs, and Poincaré and clearly put in evidence that the difficulties were all but solved. In a series of papers from 1936 to 1953, Oka introduced a brilliant collection of new ideas and systematically eliminated all difficulties. Oka's work had in itself a fruitful seed and contained the premises for the opening of wider horizons. This was quickly understood, since H. Cartan developed in his Paris seminars the algebraic basis of the theory and, about the same time, Chern and Weil set up the foundations of Hermitian and Kähler geometry of complex manifolds and vector bundles. It is at this moment that the origin of modern geometry on the complex field can be traced, which has maintained in the various streams into which it has developed its unifying characteristic of originating from complex function theory. Complex algebraic geometry, complex analytic geometry of manifolds and spaces, and complex differential geometry enjoy a fruitful permeability of methods and problems, with complex function theory always staying behind.

During the 1970s we began to believe that pressure and the needs of current research, the huge expansion of mathematical production, and the consequent fast changes of taste seemed to force most people to forget the original cultural approach, and thus that complex analysis and complex geometry were each going its own way. Our belief was corroborated by the fact that conferences, even the more traditional appointments in the field, were splitting and becoming more specialized.

In 1981 we thought that we, and perhaps other complex analysts and geometers, deserved the opportunity of having a regular appointment in which a presentation of the main results of the year would be given regardless of the specialization, and people would gather, linked by the thin thread of having used the word "complex" in their papers.

Our first experiment in 1982 had a rather enthusiastic welcome, and now we have arrived at the tenth edition. For this occasion we planned the present volume, which J. J. Kohn and L. S. Marchand kindly agreed to consider for Plenum Press. We hope that the choice of papers provides a good sample of how complex function theory is still a pervasive presence in complex geometry.

Our organizing and editing efforts would have been to no avail without the Centro Internazionale per la Ricerca Matematica (C.I.R.M.) of the Istituto Trentino di Cultura. C.I.R.M. is an independent center of mathematical research based in Trento (Italy), founded and headed by Mario Miranda. C.I.R.M. provides the scientific organizers with financial and logistic support for organizing conferences in beautiful alpine surroundings. To Mario Miranda and to C.I.R.M.'s loyal secretary, Augusto Micheletti, go our warmest thanks.

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