Elements of Continuum Mechanics and Conservation Laws

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Preface

This English edition draws on the original work of S. Godunov [5]. This book was published in Russia in 1978 and received the M. A. Lavrent'ev prize of the Russian Academy of Science in 1993. Later, the material of [5] was included in the book by both authors [7, Chapters I-IV] published in 1998. This edition also presents a new concept developed by S. Godunov on the basis of examples due to E. Romenskii. This concept was essentially developed while preparing the English translation of [7] and was intensively discussed with specialists in France, Russia, and USA last year. Four years ago we could only conjecture as to the existence of relationships between the structure of thermodynamical conservation laws and representations of the rotation group. Now, we can say more owing to the very recent results of S. Godunov and V. Gordienko (cf. Appendix and references there).

Chapters I-IV were based on lectures given by S. Godunov at the Physical Department of the Novosibirsk State University. This course was based on the original results of both authors and concerned formulations of differential equations describing high-rate deformations of metals. To study such processes, equations of elasticity should be stated in one zone, whereas equations of hydrodynamics should be used in other zones where, for example, cumulative jets appear. Plastic deformations can occur in transition zones, which causes the residual stresses. The suggested model differs from the equations of elasticity by relaxation terms simulating plastic deformations and transition of an

elastic medium to a fluid. This model was often used for numerical purposes in simulating high-rate deformations of metals [2, 6].

Using thermodynamics laws and the so-called generalized thermodynamical potentials, we can write differential equations describing physical processes in a special form. The simplest variant of such a model suggested by S. Godunov in 1961 and discussed in Sections 19–22 is well known and widely used at present [1, 3, 4, 8–10].

It should be noted that approaches to systematization of equations simulating processes in continuous media are extensively discussed in the literature. This topic is attractive for many researchers. However, the study is still far from a complete theory and there exist many different and sometimes opposite points of view, which was observed even between the authors here.

Naturally, the concept presented in the Appendix cannot be regarded as a final scheme of formalization of differential equations describing processes in continuous media but the ideas and methods expressed there could be helpful for the further study of these problems and hopefully attract the attention of not only specialists in mechanics and physics but also mathematicians studying solutions to partial differential equations and their properties.

In my opinion, the further development of the theory of thermodynamically compatible conservation laws will lead to very interesting results of general applicability. In addition, an appropriate systematization of various models of physical processes could be also useful in numerical methods

Sergei Godunov

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