Fundamentals of Plasma Physics

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Third Edition

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PREFACE

This text is intended as a general introduction to plasma physics and was designed with the main purpose of presenting a comprehensive, logical, and unified treatment of the fundamentals of plasma physics based on statistical kinetic theory. It should be useful primarily for advanced undergraduate and first-year graduate students meeting the subject of plasma physics for the first time and presupposes only a basic elementary knowledge of vector analysis, differential equations, and complex variables, as well as courses on classical mechanics and electromagnetic theory beyond sophomore level. Some effort has been made to make the book self-contained by including in the text developments of fluid mechanics and kinetic theory that are needed.

Throughout the text the emphasis is on clarity, rather than formality. The various derivations are explained in detail and, wherever possible, the physical interpretations are emphasized. The equations are presented in such a way that they connect together, without requiring the reader to do extensive algebra to bridge the gap. The features of clarity and completeness make the book suitable for self-learning and for self-paced courses.

The structure of this book is as follows. Chapter 1 consists of a basic introduction to plasma physics, at a descriptive level, intended to give the reader an overall view of the subject. The motion of charged particles under the influence of specified electric and magnetic fields is treated in detail in Chapters 2, 3, and 4. In the next five chapters the fundamental equations necessary for an elementary description of plasma

phenomena are developed. Chapter 5 introduces the concepts of phase space and distribution function, and derives the basic differential kinetic equation that governs the evolution of the distribution function in phase space. The definitions of the macroscopic variables in terms of the phase space distribution function are presented in Chapter 6 and their physical interpretations are discussed. The Maxwell-Boltzmann equilibrium distribution function is introduced in Chapter 7, as the equilibrium solution of the Boltzmann equation, and its kinetic properties are analyzed in some detail. In Chapter 8 the macroscopic transport equations for a plasma considered as a mixture of various interpenetrating fluids are derived, whereas the macroscopic transport equations for the whole plasma as a single conducting fluid are developed in Chapter 9.

The remainder of the book is devoted to applications of these basic equations in the description of a variety of important phenomena in plasmas. The problems of electrical conductivity and diffusion in plasmas are analyzed in Chapter 10, and other basic plasma phenomena, such as electron plasma oscillations and Debye shielding, are treated in Chapter 11. Simple applications of the magnetohydrodynamic equations, such as in plasma confinement by magnetic fields and the pinch effect, are presented in Chapters 12 and 13. The subject of wave phenomena in plasmas is organized in the next six chapters. A review of the basic concepts related to electromagnetic wave propagation in free space is given in Chapter 14. The propagation of very low frequency waves in a highly conducting fluid is analyzed in Chapter 15, under the title of magnetohydrodynamic waves. The various modes of wave propagation in cold and warm plasmas are considered in Chapters 16 and 17, respectively. In Chapters 18 and 19 the various properties of wave propagation in hot nonmagnetized plasmas and in hot magnetized plasmas, respectively, are analyzed. Collisional phenomena in plasmas are treated in Chapter 20, and the derivations of the Boltzmann collision integral and of the Fokker-Planck collision term are presented in Chapter 21. Finally, in Chapter 22 some applications of the Boltzmann equation to the analysis of transport phenomena in plasmas are presented.

Problems are provided at the end of each chapter, which illustrate additional applications of the theory and supplement the textual material. Most of the problems are designed in such a way as to provide a guideline for the student, including intermediate steps and answers in their statements.

The numbering of the equations, within each chapter, starts over again at each section. When reference is made to an equation using three

numbers, the first number indicates the chapter and the last two numbers indicate the section and the equation, respectively. Within the same chapter the first number is omitted. Vector quantities are represented by boldface type letters (such as $\mathbf{r}$) and unit vectors by a circumflex above the corresponding letter (such as $\hat{\mathbf{r}}$). Dyadic and triadic quantities are represented by calligraphic type letters (such as $\mathcal{Q}$).

The system of units used in this text is the rationalized MKSA. This system is based on four primary quantities: length, mass, time, and current. Its name derives from the units meter (m), kilogram (kg), second (s), and ampere (A).

The book contains more material than what can normally be covered in one semester. This permits some freedom in the selection of topics depending on the level and desired emphasis of the course, and on the interests of the students. The whole text can also be adequately covered within two semesters.

In this, as in any introductory book, the topics included clearly do not cover all areas of plasma physics. No attempt was made to present the experimental aspects of the subject. Moreover, there are some important theoretical topics that are covered only very briefly and some that have been left for more advanced courses on plasma physics, such as plasma instabilities, plasma radiation, nonlinear plasma theory, and plasma turbulence.

I am grateful to the many people who contributed to this book, both directly and indirectly, and especially to the many students to whom I had the opportunity to test my ideas in the various courses I taught over the last twenty-five years. The amount of digitalized information in a book such as this is truly enormous, and some errors may be bound to occur. Further feedback from readers will be appreciated. I wish to thank the many professors, students, and researchers who have used the first two editions of this book, all over the world, and contributed to its improvement.

J. A. Bittencourt

CONTENTS

PREFACE	v
1. INTRODUCTION	1
1. General Properties of Plasmas	1
1.1 Definition of a Plasma	1
1.2 Plasma as the Fourth State of Matter	1
1.3 Plasma Production	2
1.4 Particle Interactions and Collective Effects	3
1.5 Some Basic Plasma Phenomena	4
2. Criteria for the Definition of a Plasma	6
2.1 Macroscopic Neutrality	6
2.2 Debye Shielding	7
2.3 The Plasma Frequency	9
3. The Occurrence of Plasmas in Nature	11
3.1 The Sun and its Atmosphere	13
3.2 The Solar Wind	13
3.3 The Magnetosphere and the Van Allen Radiation Belts	14
3.4 The Ionosphere	16
3.5 Plasmas Beyond the Solar System	17
4. Applications of Plasma Physics	17
4.1 Controlled Thermonuclear Fusion	18
4.2 The Magnetohydrodynamic Generator	22
4.3 Plasma Propulsion	23
4.4 Other Plasma Devices	23

5. Theoretical Description of Plasma Phenomena	25
5.1 General Considerations on a Self-Consistent Formulation	25
5.2 Theoretical Approaches	27
Problems	28
2. CHARGED PARTICLE MOTION IN CONSTANT AND UNIFORM ELECTROMAGNETIC FIELDS	33
1. Introduction	33
2. Energy Conservation	34
3. Uniform Electrostatic Field	36
4. Uniform Magnetostatic Field	37
4.1 Formal Solution of the Equation of Motion	37
4.2 Solution in Cartesian Coordinates	40
4.3 Magnetic Moment	44
4.4 Magnetization Current	46
5. Uniform Electrostatic and Magnetostatic Fields	49
5.1 Formal Solution of the Equation of Motion	49
5.2 Solution in Cartesian Coordinates	52
6. Drift Due to an External Force	54
Problems	56
3. CHARGED PARTICLE MOTION IN NONUNIFORM MAGNETOSTATIC FIELDS	59
1. Introduction	59
2. Spatial Variation of the Magnetic Field	61
2.1 Divergence Terms	62
2.2 Gradient and Curvature Terms	64
2.3 Shear Terms	65
3. Equation of Motion in the First-Order Approximation	66
4. Average Force Over One Gyration Period	68
4.1 Parallel Force	70
4.2 Perpendicular Force	72
4.3 Total Average Force	73

5. Gradient Drift	74
6. Parallel Acceleration of the Guiding Center	74
6.1 Invariance of the Orbital Magnetic Moment and of the Magnetic Flux	75
6.2 Magnetic Mirror Effect	77
6.3 The Longitudinal Adiabatic Invariant	81
7. Curvature Drift	84
8. Combined Gradient-Curvature Drift	87
Problems	89
4. CHARGED PARTICLE MOTION IN TIME-VARYING ELECTROMAGNETIC FIELDS	95
1. Introduction	95
2. Slowly Time-Varying Electric Field	95
2.1 Equation of Motion and Polarization Drift	95
2.2 Plasma Dielectric Constant	97
3. Electric Field with Arbitrary Time Variation	100
3.1 Solution of the Equation of Motion	100
3.2 Physical Interpretation	102
3.3 Mobility Dyad	105
3.4 Plasma Conductivity Dyad	105
3.5 Cyclotron Resonance	106
4. Time-Varying Magnetic Field and Space-Varying Electric Field	108
4.1 Equation of Motion and Adiabatic Invariants	108
4.2 Magnetic Heating of a Plasma	112
5. Summary of Guiding Center Drifts and Current Densities	115
5.1 Guiding Center Drifts	115
5.2 Current Densities	115
Problems	116

5. ELEMENTS OF PLASMA KINETIC THEORY 122

1. Introduction	122
2. Phase Space	123
2.1 Single-Particle Phase Space	123
2.2 Many-Particle Phase Space	123
2.3 Volume Elements	124
3. Distribution Function	126
4. Number Density and Average Velocity	128
5. The Boltzmann Equation	129
5.1 Collisionless Boltzmann Equation	129
5.2 Jacobian of the Transformation in Phase Space	132
5.3 Effects of Particle Interactions	133
6. Relaxation Model for the Collision Term	135
7. The Vlasov Equation	136
Problems	138

6. AVERAGE VALUES AND MACROSCOPIC VARIABLES 141

1. Average Value of a Physical Quantity	141
2. Average Velocity and Peculiar Velocity	142
3. Flux	143
4. Particle Current Density	146
5. Momentum Flow Dyad or Tensor	147
6. Pressure Dyad or Tensor	148
6.1 Concept of Pressure	148
6.2 Force per Unit Area	149
6.3 Force per Unit Volume	151
6.4 Scalar Pressure and Absolute Temperature	152
7. Heat Flow Vector	154
8. Heat Flow Triad	154
9. Total Energy Flux Triad	155

10. Higher Moments of the Distribution Function	157
Problems	157

7. THE EQUILIBRIUM STATE **161**

1. The Equilibrium State Distribution Function	161
1.1 The General Principle of Detailed Balance and Binary Collisions	162
1.2 Summation Invariants	164
1.3 Maxwell-Boltzmann Distribution Function	165
1.4 Determination of the Constant Coefficients	166
1.5 Local Maxwell-Boltzmann Distribution Function	169
2. The Most Probable Distribution	169
3. Mixture of Various Particle Species	170
4. Properties of the Maxwell-Boltzmann Distribution Function	171
4.1 Distribution of a Velocity Component	171
4.2 Distribution of Speeds	174
4.3 Mean Values Related to the Molecular Speeds	176
4.4 Distribution of Thermal Kinetic Energy	178
4.5 Random Particle Flux	178
4.6 Kinetic Pressure and Heat Flux	181
5. Equilibrium in the Presence of an External Force	181
6. Degree of Ionization in Equilibrium and the Saha Equation	184
Problems	187

8. MACROSCOPIC TRANSPORT EQUATIONS **193**

1. Moments of the Boltzmann Equation	193
2. General Transport Equation	194
3. Conservation of Mass	197
3.1 Derivation of the Continuity Equation	197
3.2 Derivation by the Method of Fluid Dynamics	198
3.3 The Collision Term	200

4. Conservation of Momentum	200
4.1 Derivation of the Equation of Motion	200
4.2 The Collision Term	203
5. Conservation of Energy	204
5.1 Derivation of the Energy Transport Equation	204
5.2 Physical Interpretation	207
5.3 Simplifying Approximations	207
6. The Cold Plasma Model	210
7. The Warm Plasma Model	211
Problems	212

9. MACROSCOPIC EQUATIONS FOR A CONDUCTING FLUID **219**

1. Macroscopic Variables for a Plasma as a Conducting Fluid	219
2. Continuity Equation	222
3. Equation of Motion	223
4. Energy Equation	224
5. Electrodynamic Equations for a Conducting Fluid	227
5.1 Maxwell Curl Equations	228
5.2 Conservation of Electric Charge	228
5.3 Generalized Ohm's Law	229
6. Simplified Magnetohydrodynamic Equations	234
Problems	236

10. PLASMA CONDUCTIVITY AND DIFFUSION **238**

1. Introduction	238
2. The Langevin Equation	238
3. Linearization of the Langevin Equation	240
4. DC Conductivity and Electron Mobility	242
4.1 Isotropic Plasma	242
4.2 Anisotropic Magnetoplasma	243

5. AC Conductivity and Electron Mobility	247
6. Conductivity with Ion Motion	249
7. Plasma as a Dielectric Medium	250
8. Free Electron Diffusion	251
9. Electron Diffusion in a Magnetic Field	254
10. Ambipolar Diffusion	256
11. Diffusion in a Fully Ionized Plasma	260
Problems	262

11. SOME BASIC PLASMA PHENOMENA 269

1. Electron Plasma Oscillations	269
2. The Debye Shielding Problem	273
3. Debye Shielding Using the Vlasov Equation	278
4. Plasma Sheath	279
4.1 Physical Mechanism	280
4.2 Electric Potential on the Wall	281
4.3 Inner Structure of the Plasma Sheath	282
5. Plasma Probe	288
Problems	291

12. SIMPLE APPLICATIONS OF MAGNETOHYDRODYNAMICS 299

1. Fundamental Equations of Magnetohydrodynamics	299
1.1 Parker Modified Momentum Equation	300
1.2 The Double Adiabatic Equations of Chew, Goldberger, and Low (CGL)	302
1.3 Special Cases of the Double Adiabatic Equations	305
1.4 Energy Integral	307
2. Magnetic Viscosity and Reynolds Number	309
3. Diffusion of Magnetic Field Lines	311

4. Freezing of Magnetic Field Lines to the Plasma	312
5. Magnetic Pressure	316
6. Isobaric Surfaces	318
7. Plasma Confinement in a Magnetic Field	319
Problems	322

13. THE PINCH EFFECT 325

1. Introduction	325
2. The Equilibrium Pinch	326
3. The Bennett Pinch	332
4. Dynamic Model of the Pinch	335
5. Instabilities in a Pinched Plasma Column	341
6. The Sausage Instability	342
7. The Kink Instability	345
8. Convex Field Configurations	346
Problems	348

14. ELECTROMAGNETIC WAVES IN FREE SPACE 351

1. The Wave Equation	351
2. Solution in Plane Waves	352
3. Harmonic Waves	354
4. Polarization	358
5. Energy Flow	363
6. Wave Packets and Group Velocity	366
Problems	370

15. MAGNETOHYDRODYNAMIC WAVES 375

1. Introduction	375
1.1 Alfvén Waves	376
1.2 Magnetosonic Waves	377
2. MHD Equations for a Compressible Nonviscous Conducting Fluid	379
2.1 Basic Equations	379
2.2 Development of an Equation for the Fluid Velocity	381
3. Propagation Perpendicular to the Magnetic Field	382
4. Propagation Parallel to the Magnetic Field	383
5. Propagation at Arbitrary Directions	384
5.1 Pure Alfvén Wave	386
5.2 Fast and Slow MHD Waves	387
5.3 Phase Velocities	387
5.4 Wave Normal Surfaces	388
6. Effect of Displacement Current	390
6.1 Basic Equations	392
6.2 Equation for the Fluid Velocity	392
6.3 Propagation Across the Magnetostatic Field	393
6.4 Propagation Along the Magnetostatic Field	393
7. Damping of MHD Waves	394
7.1 Alfvén Waves	395
7.2 Sound Waves	396
7.3 Magnetosonic Waves	396
Problems	397

16. WAVES IN COLD PLASMAS 400

1. Introduction	400
2. Basic Equations of Magnetoionic Theory	401
3. Plane Wave Solutions and Linearization	402
4. Wave Propagation in Isotropic Electron Plasmas	403
4.1 Derivation of the Dispersion Relation	403
4.2 Collisionless Plasma	405

4.3	Time-Averaged Poynting Vector	407
4.4	The Effect of Collisions	410
5.	Wave Propagation in Magnetized Cold Plasmas	413
5.1	Derivation of the Dispersion Relation	414
5.2	The Appleton-Hartree Equation	418
6.	Propagation Parallel to B_0	419
7.	Propagation Perpendicular to B_0	423
8.	Propagation at Arbitrary Directions	430
8.1	Resonances and Reflection Points	430
8.2	Wave Normal Surfaces	432
8.3	The CMA Diagram	434
9.	Some Special Wave Phenomena in Cold Plasmas	439
9.1	Atmospheric Whistlers	439
9.2	Helicons	442
9.3	Faraday Rotation	444
	Problems	447

17. WAVES IN WARM PLASMAS **453**

1.	Introduction	453
2.	Waves in a Fully Ionized Isotropic Warm Plasma	453
2.1	Derivation of the Equations for the Electron and Ion Velocities	453
2.2	Longitudinal Waves	456
2.3	Transverse Wave	458
3.	Basic Equations for Waves in a Warm Magnetoplasma	460
4.	Waves in a Warm Electron Gas in a Magnetic Field	462
4.1	Derivation of the Dispersion Relation	462
4.2	Wave Propagation Along the Magnetic Field	463
4.3	Wave Propagation Normal to the Magnetic Field	466
4.4	Wave Propagation at Arbitrary Directions	469
5.	Waves in a Fully Ionized Warm Magnetoplasma	470
5.1	Derivation of the Dispersion Relation	471
5.2	Wave Propagation Along the Magnetic Field	473
5.3	Wave Propagation Normal to the Magnetic Field	477

5.4 Wave Propagation at Arbitrary Directions	479
6. Summary	479
Problems	481
18. WAVES IN HOT ISOTROPIC PLASMAS	483
1. Introduction	483
2. Basic Equations	483
3. General Results for a Plane Wave in a Hot Isotropic Plasma	485
3.1 Perturbation Charge Density and Current Density	485
3.2 Solution of the Linearized Vlasov Equation	486
3.3 Expression for the Current Density	488
3.4 Separation into the Various Modes	489
4. Electrostatic Longitudinal Wave in a Hot Isotropic Plasma	491
4.1 Development of the Dispersion Relation	491
4.2 Limiting Case of a Cold Plasma	492
4.3 High Phase Velocity Limit	493
4.4 Dispersion Relation for Maxwellian Distribution Function	494
4.5 Landau Damping	500
5. Transverse Wave in a Hot Isotropic Plasma	503
5.1 Development of the Dispersion Relation	503
5.2 Cold Plasma Result	504
5.3 Dispersion Relation for Maxwellian Distribution Function	504
5.4 Landau Damping of the Transverse Wave	505
6. The Two-Stream Instability	506
7. Summary	508
7.1 Longitudinal Mode	508
7.2 Transverse Mode	509
Problems	510

19. WAVES IN HOT MAGNETIZED PLASMAS 515

1. Introduction	515
2. Wave Propagation Along the Magnetostatic Field in a Hot Plasma	516
2.1 Linearized Vlasov Equation	516
2.2 Solution of the Linearized Vlasov Equation	517
2.3 Perturbation Current Density	522
2.4 Separation into the Various Modes	524
2.5 Longitudinal Plasma Wave	525
2.6 Transverse Electromagnetic Waves	526
2.7 Temporal Damping of the Transverse Electromagnetic Waves	529
2.8 Cyclotron Damping of the RCP Transverse Wave	531
2.9 Instabilities in the RCP Transverse Wave	532
3. Wave Propagation Across the Magnetostatic Field in a Hot Plasma	534
3.1 Solution of the Linearized Vlasov Equation	536
3.2 Current Density and the Conductivity Tensor	538
3.3 Evaluation of the Integrals	540
3.4 Separation into the Various Modes	544
3.5 Dispersion Relations	545
3.6 The Quasistatic Mode	546
3.7 The TEM Mode	550
4. Summary	552
4.1 Propagation Along $\mathbf{B}_0$ in Hot Magnetoplasmas	552
4.2 Propagation Across $\mathbf{B}_0$ in Hot Magnetoplasmas	553
Problems	554

20. PARTICLE INTERACTIONS IN PLASMAS 560

1. Introduction	560
2. Binary Collisions	561
3. Dynamics of Binary Collisions	566
4. Evaluation of the Scattering Angle	569
4.1 Two Perfectly Elastic Hard Spheres	570

4.2	Coulomb Interaction Potential	570
5.	Cross Sections	572
5.1	Differential Scattering Cross Section	574
5.2	Total Scattering Cross Section	576
5.3	Momentum Transfer Cross Section	577
6.	Cross Sections for the Hard Sphere Model	578
6.1	Differential Scattering Cross Section	578
6.2	Total Scattering Cross Section	579
6.3	Momentum Transfer Cross Section	579
7.	Cross Sections for the Coulomb Potential	580
7.1	Differential Scattering Cross Section	580
7.2	Total Scattering Cross Section	581
7.3	Momentum Transfer Cross Section	581
8.	Screening of the Coulomb Potential	582
	Problems	586

21. THE BOLTZMANN AND THE FOKKER-PLANCK EQUATIONS 589

1.	Introduction	589
2.	The Boltzmann Equation	590
2.1	Derivation of the Boltzmann Collision Integral	590
2.2	Jacobian of the Transformation	594
2.3	Assumptions in the Derivation of the Boltzmann Collision Integral	596
2.4	Rate of Change of a Physical Quantity as a Result of Collisions	597
3.	The Boltzmann's H Function	598
3.1	Boltzmann's H Theorem	599
3.2	Analysis of Boltzmann's H Theorem	601
3.3	Maximum Entropy or Minimum H Approach for Deriving the Equilibrium Distribution Function	604
3.4	Mixture of Various Particle Species	606
4.	Boltzmann Collision Term for a Weakly Ionized Plasma	607

4.1	Spherical Harmonic Expansion of the Distribution Function	607
4.2	Approximate Expression for the Boltzmann Collision Term	609
4.3	Rate of Change of Momentum Due to Collisions	611
5.	The Fokker-Planck Equation	612
5.1	Derivation of the Fokker-Planck Collision Term	612
5.2	The Fokker-Planck Coefficients for Coulomb Interactions	616
5.3	Application to Electron-Ion Collisions	621
	Problems	621

22. TRANSPORT PROCESSES IN PLASMAS **628**

1.	Introduction	628
2.	Electric Conductivity in a Nonmagnetized Plasma	629
2.1	Solution of the Boltzmann Equation	629
2.2	Electric Current Density and Conductivity	630
2.3	Conductivity for Maxwellian Distribution Function	633
3.	Electric Conductivity in a Magnetized Plasma	634
3.1	Solution of Boltzmann Equation	634
3.2	Electric Current Density and Conductivity	637
4.	Free Diffusion	640
4.1	Perturbation Distribution Function	640
4.2	Particle Flux	641
4.3	Free Diffusion Coefficient	641
5.	Diffusion in a Magnetic Field	643
5.1	Solution of Boltzmann Equation	643
5.2	Particle Flux and Diffusion Coefficients	645
6.	Heat Flow	647
6.1	General Expression for the Heat Flow Vector	647
6.2	Thermal Conductivity for a Constant Kinetic Pressure	648
6.3	Thermal Conductivity for the Adiabatic Case	649
	Problems	650

APPENDIX A	
Useful Vector Relations	655
APPENDIX B	
Useful Relations in Cartesian and in Curvilinear Coordinates	658
APPENDIX C	
Physical Constants (MKSA)	662
APPENDIX D	
Conversion Factors for Physical Units	663
APPENDIX E	
Some Important Plasma Parameters	664
APPENDIX F	
Approximate Magnitudes in Some Typical Plasmas	667
INDEX	669