Semi-Infinite Programming

Nonconvex Optimization and Its Applications

Volume 57

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Semi-Infinite Programming

Recent Advances

Edited by

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***To Rainer Hettich and
Werner Oetli,
in memoriam***

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Preface

Semi-infinite programming (SIP) deals with optimization problems in which either the number of decision variables or the number of constraints is finite. Hence SIP occupies an intermediate position between ordinary mathematical programming and fully general optimization.

Although SIP theory is quite old -recall the classical work of Haar on linear semi-infinite systems and the John's optimality conditions for SIP problems, published in 1924 and 1948, respectively-, the term appeared by the first time in a paper of Charnes, Cooper, and Kortanek (1962) devoted to duality in linear SIP. The last author also contributed significantly to the development of the first applications of SIP in economics, game theory, and air pollution control. Gustafson and Kortanek proposed -during the firsts 1970s- the first numerical methods for the effective treatment of the SIP models arising in these applications. Two decades after its inception, the publication around 1980 of five books converted SIP in a mature and independent optimization chapter. These books were two volumes of *Lecture Notes on Mathematics* devoted to SIP -edited by Hettich (in 1979), and by Fiacco and Kortanek (in 1983), respectively-, and three monographs on linear SIP (by Tichatschke, 1981), on numerical methods in SIP and their applications to approximation problems (by Hettich and Zencke, 1982), and another one on stability in SIP (by Brosowski, 1982).

Since then many papers are published every year on the theory, methods and applications of SIP and its extensions. Consequently, SIP conferences are organized regularly in order to communicate the recent advances in the field, establishing new challenges for researchers. SIP'96 took place in Cottbus, Germany, giving rise to the book *Semi-Infinite Programming* (R. Reemtsen and J.-J. Rückmann, editors, Kluwer, 1998). The present volume is based upon the last conference of this series, SIP'99, held in Alicante, Spain, and its purpose is to update the state-of-the art in a suggestive way, bringing the powerful SIP tools close enough to the potential users in different scientific and technological fields.

The volume is divided into four parts, devoted to the history, theory, methods and applications (in the large sense of the term) of SIP and related topics, respectively. Part I contains a unique paper reviewing the first decade of SIP (1962-1972). His author, K.O. Kortanek -one of the main protagonists of this period- dedicates his paper to one of the main contributors to the development of SIP, Rainer Hettich, who sadly passed away in June 2000.

Part II contains six theoretical papers. Three of them deal with convex SIP from different perspectives: constraint qualifications and error bounds (by W. Li and I. Singer), stability of the feasible set (M.A. López and V. Vera de Serio), and regularity and optimality conditions (by A. Hassouni and W. Oettly, who also died during the preparation of this book). The remaining papers are focussed on different extensions of SIP: conic linear programming (by A. Shapiro), generalized SIP (by J.-J. Rückmann and O. Stein), and disjunctive programming (by I. Eremin).

Part III contains three works exclusively devoted to the numerical treatment of different families of SIP problems and extensions. They propose an analytic center cutting plane method for linear SIP (S.-Y. Wu, S.-Ch. Fang and Ch. Lin), two logarithmic barrier methods for convex SIP (L. Abbe), and first order methods for a class of problems which contains continuously differentiable SIP problems as a particular case (E. Polak), respectively.

Finally, Part IV includes seven contributions on the connections between SIP and different fields. Those presenting numerical experiments are related to probability and statistics (by M. Dall'Aglio), optimization under uncertainty (by T. León and E. Vercher) and the design of mechanical manipulators (by C. Guarino Lo Bianco and A. Piazzzi), respectively. Two of the remaining papers deal with semi-infinite games (both of them authored by N. Llorca, J. Sánchez-Soriano, S. Tijs, and J. Timmer). Moreover, this part contains a paper on the separation of sets in normed spaces (by M.A. Goberna, M.A. López, and S.-Y. Wu), and another one related to error bounds for information domains and experimental design (by M.I. Gusev and S.A. Romanov).

We would like to express our gratitude to all the contributors for their high-quality papers, and to the referees for their valuable reports. We are very much indebted to Dr. L. Cánovas for his help in the careful preparation of the camera-ready manuscript. Last but not least, our thanks to J.R. Martindale and the staff of Kluwer Academic Publishers for having accepted the edition of this book, and for their comprehension of the six months delay in the presentation of the manuscript. This delay is the consequence of the search of a compromise between two conflicting objectives: minimizing the publication time and maximizing the quality of the book, through successive corrections.

Alicante, May 2001.

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