Undergraduate Texts in Mathematics

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Readings in Mathematics.

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Rudolf Lidl Günter Pilz

Applied Abstract Algebra

Second Edition

With 112 illustrations

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To Pamela and Gerti

Preface

Algebra is beautiful. It is so beautiful that many people forget that algebra can be very useful as well. It is still the case that students who have studied mathematics quite often enter graduate studies or enter employment without much knowledge of the applicability of the algebraic structures they have studied.

The aim of this book is to convey to senior undergraduate students, graduate students, and lecturers/instructors the fact that concepts of abstract algebra encountered previously in a first algebra course can be used in many areas of applications. Of course, in order to apply algebra, we first need some theory which then can be applied. Hence we tried to blend the theory and its applications so that the reader can experience both parts.

This book assumes knowledge of the material covered in a course on linear algebra and, preferably, a first course in (abstract) algebra covering the basics of groups, rings, and fields, although this book will provide the necessary definitions and brief summaries of the main results that will be required from such a course in algebra.

This second edition includes major changes to the first edition, published in 1984: it contains corrections and, as we believe, substantial improvements to the first four chapters of the first edition. It includes a largely new chapter on Cryptology (Chapter 5) and an enlarged chapter on Applications of Groups (Chapter 6). An extensive Chapter 7 has been added to survey other (mostly quite recent) applications, many of which

were not included in the first edition. An interdependence chart of the material in the sections is presented below.

For a one-semester course (2–3 hours per week) on Applied Algebra or Discrete Mathematics, we recommend the following path: §§1, 2, 3, 4, 6–17, 21, 22, 23, and selected topics in Chapter 7 chosen by the instructor.

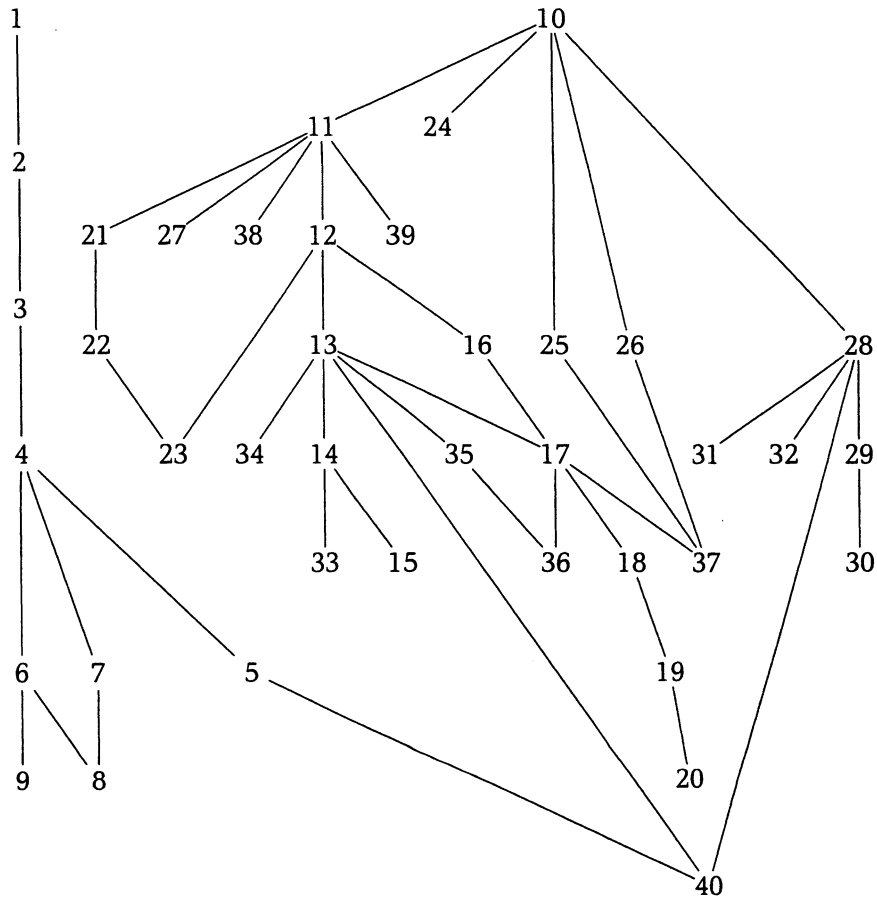
As in the first edition, we again emphasize the inclusion of worked-out examples and computational aspects in presenting the material. More than 500 exercises accompany the 40 sections. A separate solution manual for all these exercises is available from the publisher. The book also includes some historical notes and extensive references for further reading.

The text should be useful to mature mathematics students, to students in computer or information science with an interest and background knowledge in algebra, and to physical science or engineering students with a good knowledge in linear and some abstract algebra. Many of the topics covered are relevant to and have connections with computer science, computer algebra, physical sciences, and technology.

It is a great pleasure to acknowledge the assistance of colleagues and friends at various stages of preparing this second edition. Most of all, we would like to express our sincere appreciation to Franz Binder, who prepared many drafts and the final version of the entire book with LaTeX. Through his expertise in algebra, he was able to suggest many improvements and provided valuable information on many topics. Many useful suggestions and comments were provided by: E. Aichinger (Linz, Austria), G. Birkenmeier (Lafayette, Louisiana), J. Ecker (Linz, Austria), H. E. Heatherly (Lafayette, Louisiana), H. Kautschitsch (Klagenfurt, Austria), C. J. Maxson (College Station, Texas), W. B. Müller (Klagenfurt, Austria), G. L. Mullen (University Park, Pennsylvania), C. Nöbauer, P. Paule (Linz, Austria), A. P. J. Van der Walt (Stellenbosch, South Africa), and F. Winkler (Linz, Austria). Special thanks are due to L. Shevrin and I. O. Koryakov (Ekaterinenburg, Russia) for preparing a Russian translation of the first edition of our text. Their comments improved the text substantially. We also wish to thank Springer-Verlag, especially Mr. Thomas von Foerster, Mr. Steven Pisano, and Mr. Brian Howe, for their kind and patient cooperation.

October 1997 R.L. and G.P.

Interdependence Chart



Among the numerous general texts on algebra, we mention Birkhoff & MacLane (1977), Childs (1995), Herstein (1975), Jacobson (1985), and Lang (1984). Application-oriented books include Biggs (1985), Birkhoff & Bartee (1970), Bobrow & Arbib (1974), Cohen, Giusti & Mora (1996), Dorninger & Müller (1984), Fisher (1977), Gilbert (1976), Prather (1976), Preparata & Yeh (1973), Spindler (1994), and Stone (1973). A survey of the present "state of the art" in algebra is Hazewinkel (1996) (with several more volumes to follow). Historic notes on algebra can be found in Birkhoff (1976). Applications of linear algebra (which are not covered in this book) can be found in Goult (1978), Noble & Daniel (1977), Rorres & Anton (1984), and Usmani (1987). Lipschutz (1976) contains a large collection of Exercises. Good books on computational aspects ("Computer Algebra") include Geddes, Czapor & Labahn (1993), Knuth (1981), Lipson (1981), Sims (1984), Sims (1994), and Winkler (1996).

Contents

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