

Undergraduate Texts in Mathematics

Editors

S. Axler
F.W. Gehring
K.A. Ribet

Springer Science+Business Media, LLC

Undergraduate Texts in Mathematics

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Readings in Mathematics.

(continued after index)

Rudolf Lidl Günter Pilz

Applied Abstract Algebra

Second Edition

With 112 illustrations



Springer

Rudolf Lidl
DVC Office
University of Tasmania
Launceston, Tasmania 7250
Australia

Günter Pilz
Institut für Mathematik
Universität Linz
A-4040 Linz
Austria

Editorial Board

S. Axler
Mathematics Department
San Francisco State
University
San Francisco, CA 94132
USA

F.W. Gehring
Mathematics Department
East Hall
University of Michigan
Ann Arbor, MI 48109
USA

K.A. Ribet
Department of Mathematics
University of California
at Berkeley
Berkeley, CA 94720-3840
USA

Mathematics Subject Classification (1991): 05-01, 06-01, 08-01, 12-01, 13-01, 16-01, 20-01, 68-01, 93-01

Library of Congress Cataloging-in-Publication Data
Lidl, Rudolf.

Applied abstract algebra / Rudolf Lidl, Günter Pilz. — 2nd ed.
p. cm. — (Undergraduate texts in mathematics)

Includes bibliographical references and index.

1. Algebra, Abstract. I. Pilz, Günter, 1945- . II. Title.

III. Series.

QA162.L53 1997

97-22883

512'.02—dc21

Printed on acid-free paper.

© 1998 Springer Science+Business Media New York
Originally published by Springer-Verlag New York in 1998.
Softcover reprint of the hardcover 2nd edition 1998

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Production managed by Steven Pisano; manufacturing supervised by Joe Quatela.
Photocomposed by Integre Technical Publishing Co., Inc., Albuquerque, NM.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4419-3117-7 ISBN 978-1-4757-2941-2 (eBook)
DOI 10.1007/978-1-4757-2941-2

To Pamela and Gerti

Preface

Algebra is beautiful. It is so beautiful that many people forget that algebra can be very useful as well. It is still the case that students who have studied mathematics quite often enter graduate studies or enter employment without much knowledge of the applicability of the algebraic structures they have studied.

The aim of this book is to convey to senior undergraduate students, graduate students, and lecturers/instructors the fact that concepts of abstract algebra encountered previously in a first algebra course can be used in many areas of applications. Of course, in order to apply algebra, we first need some theory which then can be applied. Hence we tried to blend the theory and its applications so that the reader can experience both parts.

This book assumes knowledge of the material covered in a course on linear algebra and, preferably, a first course in (abstract) algebra covering the basics of groups, rings, and fields, although this book will provide the necessary definitions and brief summaries of the main results that will be required from such a course in algebra.

This second edition includes major changes to the first edition, published in 1984: it contains corrections and, as we believe, substantial improvements to the first four chapters of the first edition. It includes a largely new chapter on Cryptology (Chapter 5) and an enlarged chapter on Applications of Groups (Chapter 6). An extensive Chapter 7 has been added to survey other (mostly quite recent) applications, many of which

were not included in the first edition. An interdependence chart of the material in the sections is presented below.

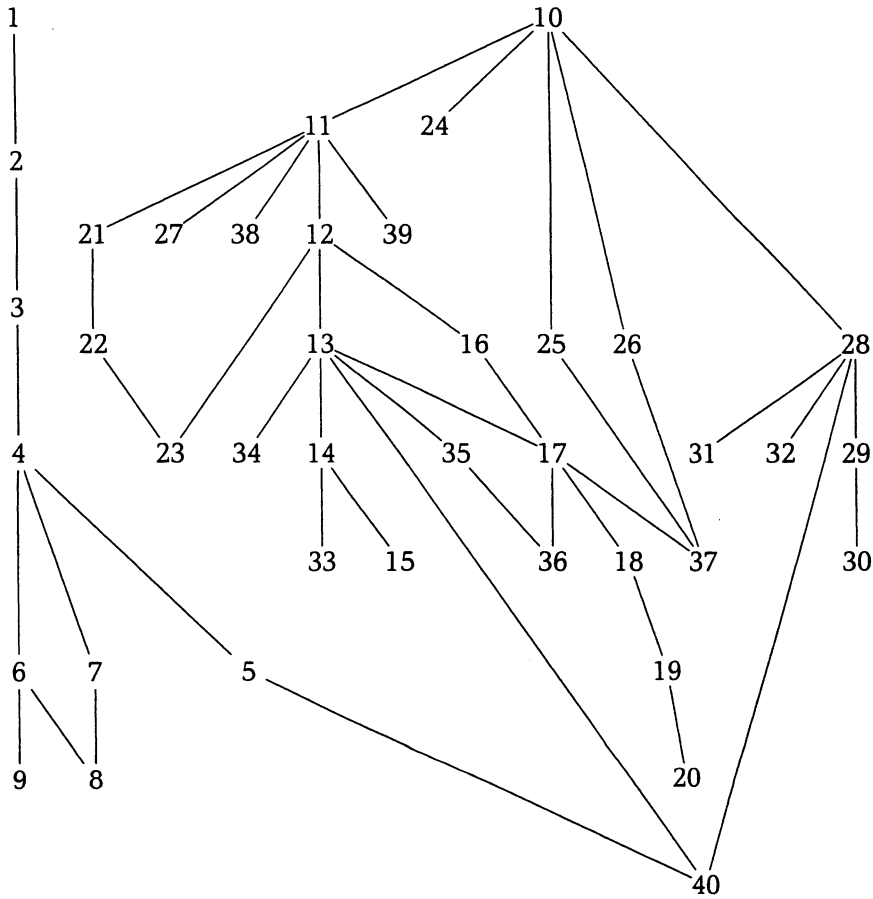
For a one-semester course (2–3 hours per week) on Applied Algebra or Discrete Mathematics, we recommend the following path: §§1, 2, 3, 4, 6–17, 21, 22, 23, and selected topics in Chapter 7 chosen by the instructor.

As in the first edition, we again emphasize the inclusion of worked-out examples and computational aspects in presenting the material. More than 500 exercises accompany the 40 sections. A separate solution manual for all these exercises is available from the publisher. The book also includes some historical notes and extensive references for further reading.

The text should be useful to mature mathematics students, to students in computer or information science with an interest and background knowledge in algebra, and to physical science or engineering students with a good knowledge in linear and some abstract algebra. Many of the topics covered are relevant to and have connections with computer science, computer algebra, physical sciences, and technology.

It is a great pleasure to acknowledge the assistance of colleagues and friends at various stages of preparing this second edition. Most of all, we would like to express our sincere appreciation to Franz Binder, who prepared many drafts and the final version of the entire book with \LaTeX . Through his expertise in algebra, he was able to suggest many improvements and provided valuable information on many topics. Many useful suggestions and comments were provided by: E. Aichinger (Linz, Austria), G. Birkenmeier (Lafayette, Louisiana), J. Ecker (Linz, Austria), H. E. Heatherly (Lafayette, Louisiana), H. Kautschitsch (Klagenfurt, Austria), C. J. Maxson (College Station, Texas), W. B. Müller (Klagenfurt, Austria), G. L. Mullen (University Park, Pennsylvania), C. Nöbauer, P. Paule (Linz, Austria), A. P. J. Van der Walt (Stellenbosch, South Africa), and F. Winkler (Linz, Austria). Special thanks are due to L. Shevrin and I. O. Koryakov (Ekaterinenburg, Russia) for preparing a Russian translation of the first edition of our text. Their comments improved the text substantially. We also wish to thank Springer-Verlag, especially Mr. Thomas von Foerster, Mr. Steven Pisano, and Mr. Brian Howe, for their kind and patient cooperation.

Interdependence Chart



Among the numerous general texts on algebra, we mention Birkhoff & MacLane (1977), Childs (1995), Herstein (1975), Jacobson (1985), and Lang (1984). Application-oriented books include Biggs (1985), Birkhoff & Bartee (1970), Bobrow & Arbib (1974), Cohen, Giusti & Mora (1996), Dorninger & Müller (1984), Fisher (1977), Gilbert (1976), Prather (1976), Preparata & Yeh (1973), Spindler (1994), and Stone (1973). A survey of the present “state of the art” in algebra is Hazewinkel (1996) (with several more volumes to follow). Historic notes on algebra can be found in Birkhoff (1976). Applications of linear algebra (which are not covered in this book) can be found in Goult (1978), Noble & Daniel (1977), Rorres & Anton (1984), and Usmani (1987). Lipschutz (1976) contains a large collection of Exercises. Good books on computational aspects (“Computer Algebra”) include Geddes, Czapor & Labahn (1993), Knuth (1981), Lipson (1981), Sims (1984), Sims (1994), and Winkler (1996).

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List of Symbols

$a R b, a \not R b$	3	$(\text{GL}(n, \mathbb{R}), \cdot)$	97	R^X	110
$[a]$	3	D_n	97	$R_1 \oplus R_2$	114
$\mathcal{P}(S)$	3	$S \leq G$	99	$\bigoplus_{i=1}^n R_i$	114
\mathbb{N}	8	$\langle X \rangle$	99, 335	$R[x]$	114
\mathbb{B}	20	$G = G_1 \oplus G_2$	100	$R[[x]]$	114
\cong_b	21	$\bigoplus_{i \in I} G_i$	100	$\deg p$	115
P_n	26	$\prod_{i \in I} G_i$	100	$\gcd(f, g)$	116
$P_n(B)$	27	$G \hookrightarrow G'$	100	\bar{p}	119
\bar{p}_B	26	$\text{Ker } f$	101	$P(R)$	119
N_d	29	$\text{Im } f$	101	$R[x, y]$	122
$F_n(B)$	24	\mathbb{Z}_n	102	$R(x)$	125
$I \trianglelefteq B$	34	\cong_n	102	$R\langle x \rangle$	126
$\ker h$	101	$N \trianglelefteq G$	103	$F(A), F(a)$	129
b	99	$I \trianglelefteq R$	112	$F(a_1, \dots, a_n)$	129
$p \rightarrow q$	81	$[G : N]$	104	$[K : F]$	130
$C \triangle B$	83	$C(G)$	107	\mathbb{F}_{p^n}	140
(G, \circ)	96	G_n	108	$\text{ind}_\beta(b)$	142
$\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, n\mathbb{Z}$	96	$(R, +, \cdot)$	109	\mathbb{F}_{p^∞}	143
S_X, S_n	97	$M_n(R)$	110	$\varphi(n)$	144

Q_n	145	$\text{Stab}(x)$	292	$S \wr T$	347
$\mu(n)$	147	$\text{Fix}(g)$	294	\rightarrow	351
$\log_\beta(b)$	142	$Z(G)$	296	\Rightarrow	351
$\text{Tr}_{F/K}(\alpha)$	150	\mathbf{R}_θ	306	$L(\mathcal{G})$	352
m_α	154	\mathbf{S}_θ	307	$W(\mathcal{A})$	353
C_s	156	$S(M)$	314	$S(\mathcal{G})$	361
d_{\min}	188	$O_2(\mathbb{R})$	315	\hat{p}	382
$\lfloor x \rfloor$	189	$S((M_i)_{i \in I})$	316	$\mathbf{D}_n, \mathbf{D}_{n,\omega}$	383
$A(n, d)$	190	$F[G]$	320	$\hat{\mathbf{a}}$	383
$\text{mld}(\mathbf{H})$	196	χ_i	321	$F\mathbb{Z}_n$	387
$S(\mathbf{y})$	199	G_S	335	$a *_t b$	403
V_n	206	\mathbb{G}_n	336	$\mathbf{H}_0, \mathbf{H}_1, \dots$	415
C^\perp, h^\perp	212	$R \diamond S$	337	\mathbf{S}_n	418
D_n	217	R^t	337	\mathbf{J}_n	418
$\lambda(n)$	262	A_*	338	$RM(m)$	419
$J(a, n)$	266	$[X, R]$	339	$P(\mathbf{H})$	423
$\lceil x \rceil$	285	X^*	340	$[\mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{K}]$	435
$P(n)$	289	M_S	345	Φ_{t_1, t_2}	437
$\text{sign}(\pi)$	290	$\mathcal{A}_1 \times \mathcal{A}_2$	346	$\mathfrak{R}(f)$	439
A_n	290	$\mathcal{A}_1 \vdash \mathcal{A}_2$	346	$\mathcal{K}(\tau)$	448
$x \sim_G y$	292	$S_1 \mid S_2$	347	\mathcal{K}_E	452
$\text{Orb}(x)$	292	$\mathcal{A}_1 \mid \mathcal{A}_2$	347	$\text{Th}(\mathcal{K})$	452