INDEPENDENT COMPONENT ANALYSIS

THEORY AND APPLICATIONS

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by

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This book is dedicated to my
parents Jeong-Bog Lee and
Sun-Ja Kang, my sister
Soon-Hie and my brother
Yu-Won.

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Abstract

Independent Component Analysis (ICA) is a signal processing method to extract independent sources given only observed data that are mixtures of the unknown sources. Recently, blind source separation by ICA has received attention because of its potential signal processing applications such as speech enhancement systems, telecommunications and medical signal processing.

This book presents theories and applications of ICA. Based on theories in probabilistic models, information theory and artificial neural networks several unsupervised learning algorithms are presented that can perform ICA. The seemingly different theories such as infomax, maximum likelihood estimation, negentropy maximization, nonlinear PCA, Busgang algorithm and cumulant-based methods are reviewed and put in an information theoretic framework to unify several lines of ICA research (Lee et al., 1998a). An extension of the infomax algorithm of Bell and Sejnowski (1995) is presented that is able to blindly separate mixed signals with sub- and super-Gaussian source distributions (Girolami, 1997b; Lee et al., 1998b). The learning algorithms are furthermore extended to deal with the multichannel blind deconvolution problem. The use of filters allows the separation of voices recorded in a real environment (cocktail party problem). Several constraints in the ICA formulation such as the linear model assumption, the number of sensors and the low-noise assumption, are tackled with new methods. In particular, an overcomplete representation of the ICA formulation (Lewicki and Sejnowski, 1998c) which includes an additive noise model can be used to infer more sources than sensors.

The second part of the book presents signal processing applications of ICA to real-world problems. The ICA algorithm has been successfully applied to many biomedical signal processing problems such as the analysis of electroencephalographic (EEG) data (Makeig et al., 1997; Jung et al., 1998a) and functional magnetic resonance imaging (fMRI) data (McKeown et al., 1998b). Bell and Sejnowski (1997) suggested that independent components of natural scenes are edge filters. Those independent image components can be used as features in pattern classification problems such as visual lip-reading and face recognition systems. The ICA algorithm can be furthermore embedded in an expectation maximization framework with the goal to classify clusters of ICA models. This approach is an extension of the Gaussian mixture model for non-Gaussian priors. Results on classification benchmarks demonstrate that ICA cluster models can improve classification results.

ICA is a fairly new and a generally applicable method to several theoretical and practical challenges in signal processing. Successful results in EEG, fMRI, speech enhancement and face recognition systems demonstrate its power and its wide range of potential applications.

Preface

At a meeting held on April 13-16, 1986 in Snowbird Utah on Neural Networks for Computing Jeanny Herault and Christian Jutten (Herault and Jutten, 1986) contributed a research paper entitled "Space or time adaptive signal processing by neural network models". They presented a recurrent neural network model and a learning algorithm based on a version of the Hebb learning rule that, they claimed, was able to blindly separate mixtures of independent signals. They demonstrated the separation of two mixed signals and also mentioned the possibility of unmixing stereoscopic visual signals with four mixtures. This paper opens a remarkable chapter in the history of signal processing, a chapter that is hardly more than 10 years old.

The problem of source separation is an old one in electrical engineering and has been well studied; many algorithms exist depending on the nature of the mixed signals. The problem of blind source separation is more difficult since without knowledge of the signals that have been mixed, it is not possible to design appropriate preprocessing to optimally separate them. The only assumption made by Herault and Jutten was independence, but additional constraints are needed on the probability distribution of the sources. If one assumes, as is often done, that the source signals are Gaussian, then it is easy to show that this problem has no general solution. Subsequent research has shown that the best performance was obtained by the Herault-Jutten network when the source signals were sub-Gaussian (Cohen and Andreou, 1992); that is, for signals whose kurtosis was less than that of a Gaussian distribution.

In the neural network field, this network model was overshadowed at the time by the more popular Hopfield network, which would soon be eclipsed in popularity by the backpropagation algorithm for multilayer perceptrons. Nonetheless, a line of research was begun that only gradually made clear the true nature of the problem. As is often the case, what is important is not the specifics of the algorithm, but the way the problem is formulated. The general framework for independent component analysis introduced by Herault and Jutten is most clearly stated in Comon (1994). Within the signal processing community, a cornucopia of ever more sophisticated algorithms was developed based on cumulants, generalizing the third-order nonlinearity first used by Herault and Jutten.

By 1994 the forefront of the neural network field had moved from supervised learning algorithms to unsupervised learning. A fast and efficient ICA algorithm was needed that could scale up with the size of the problem at least as well as backpropagation, which by this time was being used on networks with over a million weights. Anthony Bell in my laboratory was working on an infomax (Linsker, 1992) approach to ICA. Tony's first results were obtained using Mathematica and a version of his algorithm that depended on inverting a matrix (Bell and Sejnowski, 1995). This was probably fortunate since the long pauses during convergence gave him ample time to think about the problem and to benefit from vigorous discussions with Nicol Schraudolph and Paul Viola, who at the time were sharing an office with a wonderful view of the Pacific Ocean. Both Nici and Paul were working on problems that involved estimating entropy gradients, so there was a keen competition to see whose algorithm would perform best. During this period, Tony collaborated by long-distance with Te-Won Lee, who at the time was visiting Carnegie- Mellon University, on blind source separation of acoustically recorded sound mixtures, taking into account time delays.

Amari (1997a) soon realized that the infomax ICA algorithm could be improved by using the natural gradient, which multiplies the gradient of the feedforward weight matrix $\mathbf{W}$ by a positive definite matrix $\mathbf{W}^T \mathbf{W}$, and speeds up the convergence by eliminating the matrix inversion. This improvement, which was independently discovered by Cardoso and Laheld (1996), allows infomax ICA to be scaled up and makes it a practical algorithm for a variety of real-world problems. However, the original infomax ICA algorithm was only suitable for super-Gaussian sources. Te-Won Lee realized that a key to generalizing the infomax algorithm to arbitrary non-Gaussian sources was to estimate moments of the source signals and to switch the algorithm appropriately. In collaboration with Mark Girolami, who had been working on similar algorithms in the context of projection pursuit, he soon developed an efficient extended version of the infomax ICA algorithm (Lee, Girolami and Sejnowski, 1998) that is suitable for general non-Gaussian signals. All of these developments are presented in this book with a clarity that makes them accessible to anyone with an interest in ICA.

Another important component of this book is the comparison between several different approaches that have been taken to blind source separation, which include maximum likelihood, Bussgang methods based on cumulants, and projections pursuit and negentropy methods. Te-Won Lee shows that these are all closely related to the infomax framework (Lee, Girolami, Bell and Sejnowski, 1998). Thus, a large number of researchers who have attacked ICA from a variety of different directions are converging on a common set of principles and, ultimately, a well understood class of algorithms. There is still much work that is left to do. It is still true as Herault and Jutten mention in their 1986 paper, "We cannot prove convergence of this algorithm because of nonlinearity of the adaptation law and nonstationarity of the signals." We still do not have an adequate explanation for why ICA does converge for so many problems, almost always to the same solutions, even when the signals were not derived from independent sources.

What makes this book especially valuable to the practitioner is that it also includes examples of several real-world applications. Although the blind separation of mixtures of prerecorded signals is a useful benchmark, a more challenging problem is to apply ICA to recordings of real-world signals for which the underlying sources, if any, are unknown. An important example is the application of extended infomax ICA to electroencephalographic (EEG) recordings of scalp potentials in humans. The electrical signals originating from the brain are quite weak at the scalp, in the microvolt range, and there are larger artifactual components arising from eye movements and muscles. It has been a difficult challenge to eliminate these artifacts without altering the brain signals. ICA is ideally suited to this task, since the brain and the scalp are good volume conductors and to a good approximation, the recordings are different linear mixtures of the brain signals and the artifacts. The extended infomax ICA algorithm has proven to be the best method yet for separating out these artifacts, which include sub-Gaussian sources such as 60 Hz line noise and blinks, from the brain signals, which are generally super-Gaussian (Jung et al., 1998a). The future of this algorithm looks quite bright for biomedical applications, including the analysis of extremely large datasets from functional Magnetic Resonance Imaging (fMRI) experiments (McKeown et al., 1998b).

ICA can be applied to many problems where mixtures are not orthogonal and the source signals are not Gaussian. Most information bearing signals have these characteristics. There are many interesting theoretical problems in ICA that have yet to be solved and there are many new applications, such as data mining, that have yet to be explored. The appearance of this book marks an important milestone in the maturation of ICA research. The theoretical framework developed here should provide a strong foundation for future research and applications.

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I was privileged to stay at the Salk Institute, the computational neurobiology lab. (CNL), and to work with the leading researchers in biomedical signal processing: Tzyy-Ping Jung, Martin McKeown and Scott Makeig. The combined force of deep theoreticians and responsible practical researchers formed the dynamite *symmetrically doubled*-ICA team.

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Abbreviations and Symbols

Abbreviations

ARMA	autoregressive moving average
BCM	Bienenstock, Copper and Moore theory of synaptic plasticity
BSS	blind source separation
CDMA	code division multiple access
c.d.f.	cumulative density function
c.g.f.	cumulant generating function
DCA	dynamic component analysis
EEG	electroencephalographic
EOG	electrooculographic
ERP	event-related potentials
FIR	finite impuls response
fMRI	functional magnetic resonance imaging
ICA	independent component analysis
IIR	infinite impuls response
iff	if and only if
infomax	principle of information maximization preservation
KL	Kullback-Leibler
MAP	maximum a posteriori
MEG	magnetoencephalography
m.g.f.	moment generation function
MLE	maximum likelihood estimation
PCA	principal component analysis
PET	positron emission tomography
p.d.f.	probability density function
QPSK	quadrature phase shift keying
RV	random variable
SOM	self-organizing maps
SOFM	self-organizing feature maps
SVD	singular value decomposition

TDD time-delayed decorrelation

Symbols

Vectors, matrices and high-dimensional matrices are written in boldface. Usually vectors are lower-case type and matrices are higher-case type.

A	mixing matrix with elements a_{ij}
A (z)	mixing matrix of filters with elements $A_{ij}(z)$
A_x	set of possible outcomes $\{a_1, a_2, \dots\}$
a	basis functions of A (columns of A)
b	bias term
C	covariance matrix
$C(\cdot)$	cost function
c_i	i-th order cumulant
D	diagonal matrix
$D(\cdot\ \cdot)$	Kullback-Leibler divergence
D_{ij}	mixing time-delay from channel i to channel j
d_{ij}	unmixing time-delay from channel i to channel j
d_i, r_i	inverse of scale d_i and slope r_i parameters for inverting the nonlinear transfer function
E	matrix of which columns are eigenvectors of the covariance matrix
$E\{\cdot\}$	expected value
$e(\cdot)^2$	mean-squared error
$g(\cdot)$	nonlinear activation function
$f(\cdot)$	an arbitrary function
f_{jk}	coefficients of the polynomial expansion of the transfer function
	nonlinearity
$H(\cdot)$	differential entropy
h_i	orthogonal hermite polynomials
$h(\cdot)$	nonlinear inverse transfer function
h_{ik}	coefficients of the polynomial expansion of the flexible nonlinearity
i_s	information symbols
I	identity matrix
$I(\cdot)$	mutual information
$J(\cdot)$	negentropy
J	Jacobian matrix
K	number of classes
k_i	switching moments for sub- and super-Gaussian densities
K	diagonal matrix with elements k_i
$L(\cdot)$	log-likelihood function
M	number of sources
m_i	i-th order central moment
N	number of sensors
N_X	number of elements in X

$N(a, b)$	Gaussian distribution with mean a and variance b
n	white Gaussian noise signal
$O(\cdot)$	objective function
$\mathbf{P}$	performance matrix
$P(\cdot)$	Probability
$p(\cdot)$	probability density function
P	P^{th} -order of the nonlinearity mixing model
Q	Q^{th} -order of the flexible nonlinearity unmixing model
$\mathbf{R}$	permutation matrix
R_n	Lagrange's form of the remainder of a Taylor series expansion
$\mathbf{s}$	independent source signals $\mathbf{s} = [s_1(t), s_2(t), \dots, s_M(t)]^T$
$\mathbf{t}$	linear mixed signals in the nonlinear mixing model
t	time index
$T(\cdot)$	target function
$\mathbf{u}$	unmixed signals $\mathbf{u} = [u_1(t), u_2(t), \dots, u_N(t)]^T$
$\hat{\mathbf{u}}$	estimated unmixed signals
$\mathbf{V}$	whitening matrix
$\mathbf{w}$	weight vector
$\mathbf{W}$	weight matrix
$\hat{\mathbf{W}}$	weight matrix for the feedback system
X	random variable with a limited number of possible outcomes
x	continuous random variable
$\mathbf{x}$	observed vector of continuous signals $\mathbf{x} = [x_1(t), x_2(t), \dots, x_N(t)]^T$
$\mathbf{X}$	ICA mixture model: K classes of observed vectors $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_k, \dots, \mathbf{x}_K]^T$
Y	random variable with a limited number of possible outcomes
$\mathbf{y}$	neural network outputs $\mathbf{y} = [y_1(t), y_2(t), \dots, y_N(t)]^T$
$\mathbf{z}$	signals after the nonlinear unmixing $\mathbf{z} = [z_1(t), z_2(t), \dots, z_N(t)]^T$
$\text{FFT}(\cdot)$	fast fourier transform
$\text{IFFT}(\cdot)$	inverse fast fourier transform
$\text{tr}(\cdot)$	trace function
$\Delta(z)$	determinant of the matrix of filters in the frequency domain
δ_i, σ_i	scale δ_i and slope σ_i parameters of the nonlinear transfer function
ϵ	learning rate
$\mu_x(i)$	i -th order moment of x
σ^2	variance
ω	Model
Ω	set of models
$\Phi(\cdot)$	cumulant generating function
$\psi(\cdot)$	moment generating function
$\Psi(\cdot)$	higher-order polynomial terms of the Edgeworth expansion
$\varphi(\cdot)$	score function
ρ_{ij}	correlation coefficient between signals x_i and x_j .

INTRODUCTION

"Begin at the beginning," the King said, gravely, " and go on till you come to the end; then stop."

Lewis Carroll

A new star is born: ICA

Recently, blind source separation by Independent Component Analysis (ICA) has received attention because of its potential applications in signal processing such as in speech recognition systems, telecommunications and medical signal processing.

The goal of blind source separation (BSS) is to recover independent sources given only sensor observations that are linear mixtures of independent source signals. The term *blind* indicates that both the source signals and the way the signals were mixed are unknown. Independent Component Analysis (ICA) is a method for solving the blind source separation problem. It finds a linear coordinate system (the unmixing system) such that the resulting signals are statistically independent. In contrast to correlation-based transformations such as Principal Component Analysis (PCA), ICA not only decorrelates the signals (2nd-order statistics) but also reduces higher-order statistical dependencies. In other words, *ICA is a method for finding a linear non-orthogonal co-ordinate system in any multivariate data. The directions of the axes of this co-ordinate system are determined by both the second and higher order statistics of the original data. The goal is to perform a linear transform which makes the resulting variables as statistically independent from each other as possible.*

Two different research communities have considered the analysis of independent components. On one hand, the study of separating mixed sources observed in an array of sensors has been a classical and difficult signal processing problem. The seminal work on blind source separation was by Herault and Jutten (1986) where they introduced an adaptive algorithm in a simple feedback architecture that was able to separate several unknown independent sources. Their approach has been further developed by Jutten and Herault (1991), Karhunen and Joutsensalo (1994), and Cichocki et al. (1994). Comon (1994) elaborated the concept of independent component analysis and proposed cost functions related to the approximate minimization of mutual information between the sensors.

In parallel to blind source separation studies, unsupervised learning rules based on information theory were proposed by Linsker (1992). The goal was to maximize the mutual information between the inputs and outputs of a neural network. This approach is related to the principle of redundancy reduction suggested by Barlow (1961) as a coding strategy in neurons. Each neuron should encode features that are

as statistically independent as possible from other neurons over a natural ensemble of inputs; decorrelation as a strategy for visual processing was explored by Atick (1992). Nadal and Parga (1994) showed that in the low-noise case, the maximum of the mutual information between the input and output of a neural network implied that the output distribution was factorial; that is, the multivariate probability density function (p.d.f.) can be factorized as a product of marginal p.d.f.s. Roth and Baram (1996) and Bell and Sejnowski (1995) independently derived stochastic gradient learning rules for this maximization and applied them, respectively, to forecasting, time series analysis, and the blind separation of sources. Bell and Sejnowski (1995) put the blind source separation problem into an information-theoretic framework and demonstrated the separation and deconvolution of mixed sources. Their adaptive methods are more plausible from a neural processing perspective than the cumulant-based cost functions proposed Comon (1994). A similar adaptive method for source separation was proposed by Cardoso and Laheld (1996).

Other algorithms for performing ICA have been proposed from different viewpoints. Maximum Likelihood Estimation (MLE) approaches to ICA were first proposed by Gaeta and Lacoume (1990) and elaborated by Pham et al. (1992). Pearlmutter and Parra (1996), MacKay (1996) and Cardoso (1997) showed that the infomax approach of Bell and Sejnowski (1995) and the maximum likelihood estimation approach are equivalent. Girolami and Fyfe (1997c) motivated by information-theoretic indices for Exploratory Projection Pursuit (EPP) used marginal negentropy as a projection index and showed that kurtosis-seeking projection pursuit will extract one of the underlying sources from a linear mixture. A multiple output EPP network was developed to allow full separation of all the underlying sources (Girolami and Fyfe, 1997b). Nonlinear PCA algorithms for ICA which have been developed by Karhunen and Joutsensalo (1994), Xu (1993) and Oja (1997) can also be viewed from the infomax principle since they approximately minimize the sum of squares of the fourth-order marginal cumulants (Comon, 1994) and therefore approximately minimize the mutual information of the network outputs (Girolami and Fyfe, 1997d). Bell and Sejnowski (1995) have pointed out a similarity between their infomax algorithm and the Bussgang algorithm in signal processing and Lambert (1996) elucidated the connection between three different Bussgang cost functions. Lee et al. (1998a) show how the Bussgang property relates to the infomax principle and how all of these seemingly different approaches can be put into a unifying framework for the source separation problem based on an information theoretic approach.

The original infomax learning rule for blind separation by Bell and Sejnowski (1995) was suitable for super-Gaussian sources. An extension of the infomax algorithm of Bell and Sejnowski (1995) is presented in Lee et al. (1998b) that is able to blindly separate mixed signals with sub- and super-Gaussian source distributions. This was achieved by using a simple type of learning rule first derived by Girolami (1997b) by choosing negentropy as a projection pursuit index. Parameterized probability distributions that have sub- and super-Gaussian regimes were used to derive a general learning rule that preserves the simple architecture proposed by Bell and Sejnowski (1995), is optimized using the natural gradient by Amari (1998), and

uses the stability analysis of Cardoso and Laheld (1996) to switch between sub- and super-Gaussian regimes.

Extensive simulations have been performed to demonstrate the power of the learning algorithm. However, instantaneous mixing and unmixing simulations are *toy* problems and the challenge lies in dealing with real world data. Makeig et al. (1996) have applied the original infomax algorithm to EEG and ERP data showing that the algorithm can extract EEG activations and isolate artifacts. Jung et al. (1998a) show that the extended infomax algorithm is able to linearly decompose EEG artifacts such as line noise, eye blinks, and cardiac noise into independent components with sub- and super-Gaussian distributions. McKeown et al. (1998b) have used the extended ICA algorithm to investigate task-related human brain activity in fMRI data. By determining the brain regions that contained significant amounts of specific temporally independent components, they were able to specify the spatial distribution of transiently task-related brain activations. Other potential applications may result from exploring independent features in natural images. Bell and Sejnowski (1997) suggest that independent components of natural scenes are edge filters. The filters are localized, mostly oriented and similar to Gabor like filters. The outputs of the ICA filters are sparsely distributed. Bartlett and Sejnowski (1997) and Gray et al. (1998) demonstrate the successful use of the ICA filters as features in face recognition tasks and lipreading tasks respectively. In a similar manner, Bell and Sejnowski (1996) applied the infomax algorithm to learning higher-order structure of a natural sound.

For these applications, the instantaneous mixing model may be appropriate because the propagation delays are negligible. However, in real environments substantial time-delays may occur and an architecture and algorithm is needed to account for the mixing of time-delayed sources and convolved sources. The multichannel blind source separation problem has been addressed by Yellin and Weinstein (1994) and Nguyen-Thi and Jutten (1995) and others based on 4th-order cumulants criteria. An extension to time-delays and convolved sources from the infomax viewpoint using a feedback architecture was developed by Torkkola (1996a). Lee et al. (1997a) extended the blind source separation problem to a full feedback system and a full feedforward system. The feedforward architecture allows the inversion of non-minimum phase systems. In addition, the rules are extended using polynomial filter matrix algebra in the frequency domain (Lambert, 1996). The proposed method can successfully separate voices and music recorded in a real environment. Lee et al. (1997b) showed that the recognition rate of an automatic speech recognition system was increased after separating the speech signals.

Since ICA is restricted and relies on several assumptions researchers have started to tackle a few limitations of ICA. One obvious but non-trivial extension is the nonlinear mixing model. In (Hermann and Yang, 1996; Lin and Cowan, 1997; Pajunen, 1996) nonlinear components are extracted using self-organizing-feature-maps (SOFM). Other researchers (Burel, 1992; Lee et al., 1997c; Taleb and Jutten, 1997; Yang et al., 1997; Hochreiter and Schmidhuber, 1998) have used a more direct extension to the previously presented ICA models. They include certain flexible nonlinearities in the mixing model and the goal is to invert the linear mixing matrix

as well as the nonlinear mixing. More recently, Hochreiter and Schmidhuber (1998) have proposed low complexity coding and decoding approaches for nonlinear ICA. Another limitation is the under-determined problem in ICA, i.e. having less sensors than sources. Lee et al. (1998c) demonstrated that an overcomplete representation (Lewicki and Sejnowski, 1998b) of the data can be used to learn non-square mixing matrices and to infer more sources than sensors. The overcomplete framework also allows additive noise in the ICA model and can therefore be used to separate noisy mixtures.

There is now a substantial amount of literature on ICA and BSS. Reviews of the different theories can be found in Cardoso and Comon (1996); Cardoso (1998b); Lee et al. (1998a) and Nadal and Parga (1997). Several neural network learning rules are reviewed and discussed by Karhunen (1996); Cichocki and Unbehauen (1996) and Karhunen et al. (1997a).

ICA is a fairly new and a generally applicable method to several challenges in signal processing. It reveals a diversity of theoretical questions and opens a variety of potential applications. Successful results in EEG, fMRI, speech recognition and face recognition systems indicate the power and optimistic expectations in the new paradigm.

Organization of the book

This book is partitioned into Theory and Applications of ICA. The theory part of ICA includes (basic theory (chapter 1), ICA (chapter 2), unifying approach for ICA (chapter 3), multichannel deconvolution (chapter 4), overcomplete ICA (chapter 5) and nonlinear ICA (chapter 6).

- Chapter 1 starts with an introduction to Bayesian probability theory, information theory, artificial neural networks and higher-order statistics. Only some basics and some properties that are needed are recalled to further derive and explain the methods and algorithms performing ICA.
- Chapter 2 states the ICA problem and explains why decorrelation-based methods fail to separate sources and why higher-order methods are needed to solve this problem. The unsupervised learning algorithm by Bell and Sejnowski (1995) is analyzed that is able to blindly separate mixed sources with super-Gaussian distributions. An extension of this algorithm is presented that is able to blindly separate mixed signals with sub- and super-Gaussian source distributions (Giro-lami, 1997b; Lee et al., 1998b).
- Chapter 3 presents different theories recently proposed for ICA and show how they lead to the same iterative learning algorithm for blind separation of mixed independent sources. Those seemingly different theories are reviewed and put in an information theoretic framework to unify several lines of research.
- Chapter 4 deals with time-delayed and convolved sources, the multichannel blind deconvolution problem. Learning algorithms for the feedforward and feedback

architecture are presented. These methods is applied to multisensory recordings and is able to separate voices and music recorded in real environments.

- Chapter 5 presents ICA results using an overcomplete representation (Lewicki and Sejnowski, 1998b). This generalization includes an additive noise model and allows the separation of more sources than mixtures.
- Chapter 6 tries to eliminate several constraints in the standard ICA formulation. The linear model is generalized by simple nonlinear mixing models for nonlinear ICA. A set of learning algorithms are derived and verified in simulations.

The applications part of ICA includes ICA for biomedical signal processing (chapter 7), feature extraction (chapter 8) and ICA for unsupervised classification (chapter 9).

- Chapter 7 presents examples on how ICA can be used to (1) isolate artifacts in EEG recordings and (2) to detect transiently activated brain signals in fMRI experiments.
- Chapter 8 demonstrates how ICA can extract features from natural images. Those features can be used in recognition systems such as face recognition and lipreading systems to improve overall performance.
- Chapter 9 shows how the ICA algorithm can be further embedded in an expectation maximization framework with the goal to classify mixtures of ICA models. This approach is an extension of the Gaussian mixture model for non-Gaussian priors. Results on several classification benchmarks demonstrate that ICA cluster models can improve many classification results.

Chapter 10 gives conclusions by summarizing the main results and discussing future challenges in ICA research.