# STOCHASTIC DOMINANCE:

## Investment Decision Making Under Uncertainty

**Stochastic Dominance**

by Haim Levy
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*Printed on acid-free paper.*

*To Tal, Shira, Tamar, and Neta*

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## PREFACE

This book is devoted to investment decision-making under uncertainty. The book covers three basic approaches to this process:

a) The stochastic dominance approach, developed on the foundation of von-Neumann and Morgenstern[1] expected utility paradigm.

b) The mean-variance approach developed by Markowitz[2] on the foundation of von-Neumann and Morgenstern's expected utility or simply on the assumption of a utility function based on mean and variance.

c) The non-expected utility approach, focusing on prospect theory and its modified version, cumulative prospect theory. This theory is based on an experimental finding that subjects participating in laboratory experiments often violate expected utility maximization: They tend to use subjective probability beliefs that differ systematically from the objective probabilities and to base their decisions on changes in wealth rather than on total wealth.

The above approaches are discussed and compared in this book. We also discuss cases in which stochastic dominance rules coincide with the mean-variance rule and cases in which contradictions between these two approaches may occur. We then discuss the relationship between stochastic dominance rules and prospect theory, and establish a new investment decision rule which combines the two and which we call prospect stochastic dominance. Although all three approaches are discussed, most of the book is devoted to the stochastic dominance paradigm.

Concepts similar to stochastic dominance have been known for many years but the two papers published by Hadar and Russell, and Hanoch and Levy in 1969, and the paper published by Rochschild and Stiglitz in 1970[3] paved the way for a new paradigm called stochastic dominance, with hundreds of studies following in their tracks. These studies, deal with theoretical as well as empirical issues in various areas of economics, finance, accounting, statistics, agriculture, and medicine.

The need to develop the stochastic dominance rules, at least in the view of the author of this book, stems from paradoxes that are sometimes revealed by the commonly

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[1] von Neumann, J., and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, N.J., 1953.

[2] Markowitz, H.M., "Portfolio Selection," *Journal of Finance*, 1952, pp. 77–91.

[3] Hadar, J. and W.R. Russell, "Rules for Ordering Uncertain Prospects," *American Economic Review*, 1969, pp. 25–34; Hanoch, G. and H. Levy, "The Efficiency Analysis of Choices Involving Risk," *Review of Economic Studies*, 1969, pp. 335–346 and Rothschild M. and J. Stiglitz, "Increasing Risk. I. A Definition," *Journal of Economic Theory*, 1970, pp. 225–243.

used mean-variance rule. To be more specific, there are cases in which a clear-cut choice between two risky assets exist, yet the mean-variance rule is unable to distinguish between the two alternate investments. When I was a second-year MBA student, only the mean-variance investment rule was taught. I presented my teacher with the case of two alternative investments: x providing $1 or $2 with equal probability and y providing $2 or $4 with equal probability, with an identical initial investment of, say, $1.1. A simple calculation shows that both the mean and the variance of y are greater than the corresponding parameters of x; hence the mean-variance rule remains silent regarding the choice between x and y. Yet, any rational investor would (and should) select y, because the lowest return on y is equal to the largest return on x. Well, this is a trivial case in which the mean-variance rule fails to show the superiority of one investment over another. However, there are many more such cases in which the mean-variance rule is unable to distinguish between two investments. These cases are sometimes quite complex and the superiority of one investment over the other cannot be detected by the naked eye: Hence my motivation to develop general decision rules, well-known nowadays as stochastic dominance rules.

In spite of its common use, the mean-variance rule coincides precisely[4] with the expected utility paradigm only in two cases: normal distribution of returns in the face of risk aversion, and quadratic utility function. The quadratic utility function, apart from the disadvantage of restricting the analysis to one type of preference, has other drawbacks, too. Hence, most researchers focus on the normal case. However, in the normal case, we face a severe problem: the returns range is from minus infinity to plus infinity but actual rates of returns are bounded by –100%, that is, the asset price can drop to zero and no further. Assuming lognormal distribution provides a rescue from this limited liability issue because lognormal distribution is defined only for non-negative asset prices, thus conforming with the fact that, in practice, asset prices cannot be negative. However, although the assumption of lognormal distribution overcomes the limited liability issue, it gives rise to another issue: If each asset is lognormally distributed, linear combination of these assets (a portfolio) will not be lognormally distributed. The continuous time model suggested by Merton,[5] overcomes this difficulty of lognormal distribution because, for any finite terminal date, each selected portfolio will be lognormally distributed. However, the lognormal case is saved at the cost of assuming a continuous time model in which the investor revises the portfolio weights continuously. This implies that even a tiny transaction cost will induce a negative rate of return on the terminal date; hence, existing transaction costs ruin the model.

The stochastic dominance framework does not suffer from the above deficiencies. However, this paradigm has its own deficiencies: a method to construct all the effi-

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[4] One can use the mean-variance rule as an approximation to the expected utility. For more details, see Levy, H., and H. Markowitz, "Approximating Expected Utility by a Function of Mean and Variance," *American Economic Review*, 1979, pp. 308–317.

[5] Merton, R.E., "An Intertemporal Capital Pricing Model," *Econometrica*, September 1973, pp. 867–887.

cient portfolios and a separation theorem (as in the mean-variance framework) have yet to be developed. Therefore, we suggest that stochastic dominance rules do not substitute for the mean-variance rule but rather offer an alternative approach, complimenting rather than replacing it. However, in cases that are not related to portfolio construction (e.g., applications in agriculture, medicine, statistics, and some applications in economics and finance), application of the stochastic dominance paradigm will be optimal because no assumptions are needed regarding the distribution of returns.

The book starts with various commonly used measures of risk (Chapter 1) leading up to the expected utility paradigm (Chapter 2) which shows that the only relevant measure of risk is the risk premium. As the risk premium varies from one investor to another, we conclude that, in general, no one single objective index has the capacity to rank investments by their risk. Thus, the whole distribution of returns rather than one measure of profitability and one measure of risk has to be considered. Chapter 3 forms the heart of the book. In this chapter, we develop and discuss first, second, and third degree stochastic dominance rules. In Chapter 4, we extend the stochastic dominance rules to include riskless assets. In order to do this, we first reformulate the stochastic dominance rules in terms of distribution quantiles rather than cumulative distributions. Algorithms for all these stochastic dominance rules are provided in Chapter 5. Having the general stochastic dominance rules with no constraints on the distribution returns under our belt, we proceed in Chapter 6, to stochastic dominance for specific distributions including normal, lognormal and other truncated distributions. Then, in Chapter 7, we provide empirical evidence regarding the effectiveness of the stochastic dominance rules as well as the mean-variance rule.

In Chapter 8 we present a few of the many applications of stochastic dominance rules in various fields of research. We discuss these applications briefly and provide references to these studies at the end of the book. Chapter 9 is devoted to the definition of situations in which one asset is identified as "more risky" than another asset, and the extension of this definition to DARA utility functions as well as to the case of the riskless asset. Chapter 10 is devoted to stochastic dominance and diversification and, in particular, to the effect of changes in the cumulative distributions on diversification. Chapter 11 analyzes the effect of changes in the assumed investment horizon on the efficient set in the frameworks of mean-variance and stochastic dominance.

The capital asset pricing model (CAPM) of Sharpe and Lintner[6] is undoubtedly one of the main cornerstones of modern finance. However, the CAPM holds only under a set of confining assumptions, one of them being that all investors have the same investment horizon. In Chapter 12, based on the work of Levy and Samuelson,[7] we use

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[6] Sharpe, W.F., "Capital Asset Prices: A Theory of Market Equillibrium Under Conditions of Risk," *Journal of Finance*, September, 1964, pp: 425–442, and Lintner, J., "Security Prices, Risk, and Maximal Gains from Diversification," *Journal of Finance*, December 1965, pp. 587–615.

[7] Levy, H., and P.A., Samuelson, "The Capital Asset Pricing Model with Diverse Holding Periods," *Management Science*, November 1992, pp. 1529–1542.

stochastic dominance rules to show that the CAPM holds under a much wider set of assumptions, even if investors do not have the same horizon.

Chapter 13 is devoted to non-expected utility theory with Prospect Theory suggested by Kahaneman and Tversky[8] as the main competing theory. We suggest a reconciliation between the two competing theories by focusing on short- and long-term investment decisions.

Chapter 14 concludes the book with suggestions for further research and presentation of unsolved problems in the area of investment decision making, with emphasis on stochastic dominance. Readers interested in this field are welcome to pursue these research ideas.

## AUDIENCE

This book is intended mainly for Ph.D. students, advanced MBA students specializing in finance, and advanced MA economics students interested in the economics of uncertainty. The book can be used also as a supplementary book in post-graduate courses on portfolio selection and investment decision making under uncertainty.

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[8] Kahaneman, D., and A. Tversky, "Prospect Theory of Decisions Under Risk," *Econometrica*, 1979, pp. 263–291.