**SPATIOTEMPORAL
ENVIRONMENTAL HEALTH
MODELLING:
*A Tractatus Stochasticus***

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by

George Christakos

and

Dionissios T. Hristopulos

*School of Public Health
Department of Environmental Sciences and Engineering
The University of North Carolina at Chapel Hill*

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*Dedicated to the memory of my father Panayiotis,
who gave and cared so much,
and to my mother Mary who continues to do so.*

George Christakos

*To my parents, Theodoros and Anna, and my wife, Lisa,
for their love and support;
To Greece, for all that she has given me.*

Dionissios Hristopulos

Forward

*"La pensée n'est qu'un éclair au milieu de la nuit.
Mais c'est cet éclair qui est tout".*

H. Poincaré

Protecting and promoting health at the population level requires a rigorous interdisciplinary scientific base. Health is the product of an array of complex physiological, environmental, and behavioral processes. Success in understanding and addressing these processes require the capacities of many different scientific disciplines. The science of public health has made remarkable advances in recent years as its practitioners have explored new ways of linking and applying knowledge from core disciplines in the mathematical, natural, and behavioral sciences. Environmental health is an essential element of public health science, and its concepts and methods epitomize the interdisciplinary nature of public health.

The successful integration of concepts and methods from diverse fields of study requires both creativity and depth of knowledge. At its best, such integration creates a scientific synergism that enables new insight regarding the problems and issues under study. In this book, George Christakos and Dionissios Hristopulos create just such a synergism for the practice of environmental health science. The authors explore a new and powerful approach to environmental health science --one which brings together principles from modern stochastic theory with more traditional methods of examining environmental processes and their associate health effects. This approach offers fresh views of common environmental health issues, and new insight regarding methods for assessment and control.

Public health scientists, decision-makers, and students alike have much to gain from the concepts and methods described in this book. By advancing our understanding of the natural and physiological processes which determine health, the authors make a substantial contribution to the field of public health and its mission of assuring population health. In doing so, the authors underscore the importance of interdisciplinary scientific approaches to public health issues.

William L. Roper, MD, MPH
Dean, School of Public Health
The University of North Carolina at Chapel Hill

Preface

"We haven't the money, so we've got to think."

E. Rutherford

This book is the culmination of an investigation into the applicability of modern stochastics to problems of environmental health science. The result is a stochastic interpretation of processes (physical, chemical, biological, epidemic, etc.) that account for changes in the state of the environment and human health. This interpretation exposes the inherent uncertainty in such processes, and advocates prediction in terms of spatiotemporal probability laws and statistical statements. The deterministic approach is based on the hypothesis that a complete characterization of the system is available. Deterministic theories provide inadequate representations for most environmental health processes, due to the incomplete knowledge of the underlying mechanisms and the partial characterization of model parameters. In the stochastic viewpoint the focus is shifted from the analysis of a single, perfectly determined system, to an ensemble of possible systems, which represent all the potentialities permitted by the existing knowledge of the system's properties. Thus, instead of concentrating on single system properties, the stochastic approach calculates ensemble properties. Finally, conclusions about the former are derived on the basis of the latter. Stochastic analysis can be viewed as a mathematical generalization of conventional deterministic analysis. This generalization is more flexible in practice and can handle problems in environmental health science more accurately and efficiently than the often-inadequate deterministic methods. In recent years a new synthesis has been emerging in the study of natural phenomena that combines physical models with stochastic methods, which account for uncertainties and variabilities. The new synthetic view, called modern stochastics, and its applications in environmental health science will be the focus of this book. The answers that modern stochastics supplies are not always obvious to the deterministically minded, and in certain cases deep-rooted prejudices may need adjustment.

As its title dictates, this book uses stochastic concepts and methods to build links between models and techniques of environmental sciences, on the one hand, and health sciences on the other. To our knowledge, this is a task that is not shared by any other book currently available in the literature. It is, however, a natural development, because the environment and human health are closely related and interdependent in many ways.

Rigorous quantitative analysis of the associations between, e.g., subsurface contaminant transport models and human exposure, or between space-time ozone distribution and disease incidence rates, are of great significance in health management. Such associations are crucial for site selection and human exposure assessment in epidemiologic studies. Another example is carcinogenesis, which is a random multistep process controlled both by cancer genes and environmental conditions. Separated, environmental and health models rarely lend each other assistance; only when their associations and interactions have been clearly established, they can illuminate and enrich one another.

In light of the above considerations, one may distinguish two main objectives of the book. The first is to provide a coherent and unified account of modern spatiotemporal stochastics for the environmental health scientist. At this level the book can function as a textbook as well as a reference volume, since it contains a number of novel methods and results in stochastic environmental health modelling. The second goal is to apply modern stochastics in environmental health science problems in a way that emphasizes their close relations and strong interdependence. Of course, since environmental health science is progressing very fast and in various directions, no text of any reasonable size can exhaust all the possibilities. Therefore, in this book we have chosen to focus on a number of carefully selected applications. However, modern stochastics has applications in phenomena that cover a considerably wider range than what is presented here, and practically includes every aspect of environmental health science.

Although this is definitely an application-oriented book, it contains several philosophical theses and comments on methodological issues. The reason is that, in our view, modern stochastics is not just a collection of abstract equations and techniques for problem solving. Instead, it constitutes a scientific paradigm based on a more flexible conceptualization of reality than its deterministic counterpart. Efficient use of the stochastic paradigm presupposes an appropriate conceptual framework, within which the significance and emerging applications of notions such as uncertainty, variability, heterogeneity and risk can be debated in the scientific forums. In addition, the new paradigm can help scientists develop an ability to argue coherently and convincingly on a wide range of issues, a skill which is extremely important in communicating their efforts to broader audiences in the public and political arenas.

Throughout the book our approach is conceptual, having a strong pedagogic orientation with many examples presented for edification and assimilation of the conceptual principles and the mathematical techniques. The variety of applications also provides a framework for evaluation, and a realization that there can be a systematic organization for many concepts, tools and perspectives. Furthermore, the aim of this approach has been to find quantitative

solutions to environmental health problems that are serviceable to research scientists and practitioners who work in these fields.

Based on the fact that the roots of many useful stochastic concepts and tools have multiple branches that penetrate different scientific disciplines, the book has been written with a diverse audience in mind. It should be of interest to scholars and researchers in the environmental sciences, epidemiology, health sciences, statistics, risk analysis, mathematics, and decision making. Furthermore, practitioners in the above areas will find a variety of theoretical and computational tools that should be of immediate practical use. The book is also intended as a textbook for graduate-level modelling courses in environmental modelling, health risk assessment, geostatistics, descriptive epidemiology, biostatistics, and medical geography.

In writing this book our approach has been guided by the following dictum: "True education is what survives when a person has forgotten all that he/she has been taught." The aim of this book goes beyond providing a compendium of technical recipes, black-box computer algorithms and simplified remedies for certain problems. Instead, this book aims to give aspiring scientists some motivation and tools for addressing environmental health issues with a critical mind. The book should be read in the same spirit, and we encourage the reader to question the arguments that we use, and to think of counter-examples. The book is intended to stimulate thought, not to be an alternative to it. If readers examine the material presented critically, they will discover areas where they may agree or disagree with the authors. It is our hope that in either case they will clarify their own beliefs and, perhaps, prejudices in the process.

We are grateful to several colleagues for stimulating discussions, especially to C.T. Miller, D. Crawford-Brown, P. Bogaert and G. Cassiani. Valuable contributions were made by the first author's former and current students, V.M. Vyas, B.R. Killam, J-J. Lai, X. Li, D.L. Oliver, G. Thesing, A. Kolovos, M. Serre and K-M. Choi. Finally, we greatly acknowledge funding from the National Institute of Environmental Health Sciences (Grant no. P42 ES05948-02), the Department of Energy (Grant no. DE-FC09-93SR18262), and the Army Research Office (Grant no. DAAL03-92-G-0111). Without their support, this book would have never been written, for better or for worse.

George Christakos

Dionissios Hristopulos

Chapel Hill, N.C.

Note: The following notation is used throughout the book:

- §5 denotes the 5th section in the same Chapter.
- §II.3 denotes the 3rd section of Chapter II.
- Eq. (4.2) denotes the 2nd equation of section 4 in the same Chapter.
- Eq. (IV.9.5) denotes the 5th equation of section 9 in Chapter IV. This notation is used when we refer to an equation from within a different Chapter.

Table of Contents

Chapter I: FUNDAMENTAL PRINCIPLES OF STOCHASTIC ENVIRONMENTAL HEALTH MODELLING	1
1. On the method of environmental health science.	1
2. Fundamental principles of the stochastic mode of thinking.	7
3. The notion of spatiotemporal continuity.	16
3.1 Philosophical aspects of space and time	17
3.2 The space-time continuum concept	18
3.3 Natural vs. health processes	19
3.4 Space-time coordinates	20
4. The field concept in environmental health science	22
5. Knowledge bases in the space-time domain.	26
6. Spatiotemporal scales.	27
6.1 Spatiotemporal scales for natural processes	28
6.2 Spatiotemporal scales for health processes	31
6.3 The hierarchy of scales	32
7. An overview of the environmental health paradigm	34
 Chapter II: ENVIRONMENTAL EXPOSURE FIELDS AND THEIR HEALTH EFFECTS	 37
1. The holistic environmental exposure -health effect perspective	37
2. Studying environmental exposure fields	42
2.1 Pollution sources	42
2.2 Environmental pollutants	43
2.3 Characterization of the environment	44
2.4 Environmental compartments	45
2.5 Environmental media	46
2.6 Natural fields	47
2.7 Evolution mechanisms	49
3. Human Exposure Concepts	53
3.1 Basic notions	53
3.2 Mathematical expressions for exposure	59
3.3 Mathematical expressions for biomarkers	64

3.4	Exposure as a practical biomarker estimator	68
4.	Health Effect Concepts	69
4.1	The health impact pathway and the various kinds of health effects	69
4.2	Mathematical models for health effect indicators	71
4.3	Mathematical models for individual-based and population-based dose and exposure-response curves	73
4.4	Spatiotemporal coordinates in human exposure analysis	77

Chapter III: SPATIOTEMPORAL RANDOM FIELDS IN EXPOSURE ANALYSIS AND ASSESSMENT

		81
1.	A theatrical introduction to the random field concept	81
2.	The spatiotemporal random field	83
2.1	Basic mathematical formulation	84
2.2	Characterization in terms of ensemble functions	87
3.	S/TRF classifications	90
4.	Space-time metrics	95
5.	Spatiotemporal correlation models	97
5.1	General properties	97
5.2	Criteria of permissibility	99
5.3	Useful tools for constructing spatiotemporal correlation functions	101
6.	Separable space-time covariance models	102
6.1	Separability in the space-time domain	102
6.2	Classes of spatial covariance models	103
7.	Natural models and covariance functions	109
8.	Regression S/TRF models	114
8.1	Mathematical formulation	114
8.2	Statistical inference issues	116
9.	Wave representation of S/TRF	118

Chapter IV: MODELLING EXPOSURE HETEROGENEITIES

1.	A class of spatiotemporal random fields	121
1.1	The central idea and definitions	121
1.2	Determination of the Q -operator in practice and its physical significance	125
1.3	An S/TRF decomposition	127

2. Spatiotemporal polynomial notation	127
3. Orders of spatiotemporal continuity ν/μ	129
4. Continuum representations of S/TRF- ν/μ	132
5. Discrete representations: spatiotemporal increments of orders ν/μ	135
6. Spectral Representations	139
7. Generalized spatiotemporal random fields	141
8. Permissibility criteria	147
8.1 Analysis in the continuous spatiotemporal domain - The generalized spatiotemporal covariance	148
8.2 Some examples in the discrete space-time domain	152
8.3 Permissibility conditions for polynomial covariances	154
8.4 Permissibility conditions for vector S/TRF	157
9. Generalized covariance models derived from PDE's	160
10. Space-time separability	163
11. New generalized spatiotemporal covariance models	164
11.1 The Polynomial model	164
11.2 The Exponential model	165
11.3 The Gaussian model	169

Chapter V: SPATIOTEMPORAL MAPPING OF ENVIRONMENTAL HEALTH PROCESSES - THE BME APPROACH 171

1. About spatiotemporal maps	171
2. Space-time mapping fundamentals	173
2.1 The basic epistemic framework of the BME approach	173
2.2 Other mapping approaches	175
2.3 Operational concepts of scale	176
3. Mathematical formulation of the BME approach	177
4. Some analytical results	185
4.1 Multi-point BME	185
4.2 Single-point BME	186
5. Other formulations of the BME approach	189
5.1 Functional BME	189
5.2 Vector BME	191
6. BME vs. MMSE estimates	192

Chapter VI: SPATIOTEMPORAL MMSE MAPPING	193
1. Introduction	193
2. Mathematical Formulation	194
3. Linear MMSE space-time mapping	195
3.1 Mathematical formulation	195
3.2 Recursive formulations	199
3.3 More properties	199
4. Homogeneous-Stationary S/TRF: Space-time Kriging forms	201
4.1 Biased MMSE mapping	201
4.2 Unbiased MMSE mapping	202
5. Nonhomogeneous-nonstationary S/TRF: The case of S/TRF- v/μ	206
6. Vector MMSE space-time mapping	212
7. Implementation issues - Applications	214
7.1 Practical map generation	214
7.2 Trend determination	221
8. Regression S/TRF	224
9. BME vs. MMSE estimators	226
10. BME vs. Kriging estimators	226
Chapter VII: STOCHASTIC PARTIAL DIFFERENTIAL EQUATION MODELLING OF FLOW AND TRANSPORT	231
1. Introduction	231
2. Perturbation expansions	235
3. The diagrammatic approach	240
3.1 The diagrammatic language - Physical space	241
3.2 Diagrams in frequency space	243
4. Self-consistent equations	247
4.1 The basic concepts	247
4.2 The asymptotic limit	251
5. Diagrammatic methods in subsurface hydrology	252
6. Renormalization group analysis-Change of scale	253
6.1 The basic concepts	254
6.2 Diagrammatic approximations for the self-energy	258
6.3 Frequency space RNG treatment of transport	259

6.4	Perturbative RNG schemes	261
6.5	RNG analysis of subsurface pollution transport	263
7.	The space transformation technique	265
7.1	Basic definitions	265
7.2	Some properties of STs	267
7.3	Numerical implementations of ST	270
7.4	Applications of STs in SPDE solving	272
7.5	Differential geometric approach to multiphase flow	275
7.6	Boundary effects	276

Chapter VIII: STOCHASTIC PHYSIOLOGICALLY-BASED POLLUTOKINETIC MODELLING 279

1.	Introduction	279
2.	Compartmental analysis of pollutokinetics	280
3.	One-compartment stochastic pollutokinetics	284
4.	Multi-compartment stochastic pollutokinetics	290
5.	Stochastic pollutokinetics and health effects	292
6.	Some generalizations	297

Chapter IX: STOCHASTIC EXPOSURE AND HEALTH INDICATORS 299

1.	Introduction	299
2.	Stochastic exposure indicators	302
2.1	The exposure S/TRF-pair	303
2.2	The stochastic exposure indicator concept	305
2.3	One-point stochastic exposure indicators	306
2.4	Two-point stochastic exposure indicators	309
2.5	Uncertainty in the exposure threshold level	313
2.6	Other properties of exposure indicators	315
2.7	Ergodicity and sample averaging of exposure indicators	318
3.	Stochastic cell-based health effect indicators	322
3.1	Cellular structures	322
3.2	The threshold postulate	323
3.3	Relationships between the basic cell indicators	329
3.4	Derived cell indicators	331

3.5	Scale and modelling effects	331
3.6	The multistage postulate	335
4.	Stochastic formulation of traditional population health effect indicators	340
4.1	Two fundamental health effect indicators	341
4.2	Person-time rates	346
4.3	Rate ratios	349
4.4	Spatiotemporal variation and mapping of population health indicators - The case of the North Carolina breast cancer incidence distribution	349
5.	Correlation analysis	355
5.1	Population health damage indicators	355
5.2	Linear PER model	359
5.3	Uniform population density	360
5.4	Stochastic trends and uncertainty assessment in exposure-damage models	361
5.5	Time delayed PER relationships	363
6.	Spatiotemporal maps of health damage indicators - The ozone case	363
7.	Analysis of disease - Exposure association	369
	Bibliography	375
	Index	391