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Continued at end of book

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Introduction to Time Series and Forecasting

With 122 Illustrations and Diskette

Springer

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To Pam and Patti

Preface

This book is aimed at the reader who wishes to gain a working knowledge of time series and forecasting methods as applied in economics, engineering and the natural and social sciences. Unlike our earlier book, *Time Series: Theory and Methods*, referred to in the text as TSTM, this one has as prerequisites only a knowledge of basic calculus, matrix algebra and elementary statistics at the level (for example) of Mendenhall, Wackerly and Scheaffer (1990). It is intended for upper-level undergraduate students and beginning graduate students.

The emphasis is on methods and the analysis of data sets. A time series package enabling the reader to reproduce most of the calculations in the text (and to analyze further data sets of the reader's own choosing) is included on the diskette which accompanies the book. The data sets used in the book are also included. The programs require an IBM-PC-compatible computer. Instructions for installation and running the package (under either DOS or Windows) are contained in the file README.DOC on the diskette.

Use of the computer package requires very little prior familiarity with computing. The programs are all menu-driven so that the reader can immediately apply the techniques described in the book to time series data, with a minimal investment of time in the computational and algorithmic aspects of the analysis. A step-by-step introduction to the package is given in Appendix D and some further illustrative examples are given in the file README.DOC. The book could also be used in conjunction with other computer packages for handling time series. Chapter 14 of the book by Venables and Ripley (1994) describes how to perform many of the calculations using S-plus.

There are numerous problems at the end of each chapter, many of which involve use of the programs to study the data sets provided.

To make the underlying theory accessible to a wider audience, we have stated some of the key mathematical results without proof, but have attempted to ensure that the logical structure of the development is otherwise complete. (References to proofs are provided for the interested reader.)

We have added a number of topics not considered in TSTM. These include harmonic regression (Chapter 1), the Burg and Hannan-Rissanen algorithms (Chapter 5),

unit roots in time series models and regression with ARMA errors (Chapter 6), structural models, the EM algorithm and generalized state-space models, with applications to time series of count data (Chapter 8), and the Holt-Winters and ARAR forecasting algorithms (Chapter 9). The last chapter introduces the reader to transfer function models, intervention analysis, and nonlinear (including ARCH), continuous-time, and long-memory models.

There is sufficient material here for a full-year introduction to univariate and multivariate time series and forecasting. Chapters 1 through 6 have been used for several years as an introductory one-semester course in univariate time series at Royal Melbourne Institute of Technology. The chapter on spectral analysis can be excluded without loss of continuity by readers who are so inclined.

The book has benefited greatly from comments and corrections made by readers of TSTM, users of the computer package ITSM, and readers of preliminary versions of this book. To all these we express our thanks, particularly to two reviewers for their many valuable suggestions. We are especially indebted also to Anthony Brockwell and Joe Mandarino for their contributions to the development of the computer package, to Rob Hyndman for his collaboration in the writing of the manual, ITSM for Windows, to Carey and Michele Davis for their assistance in assembling the index, to the National Science Foundation for support of the research on which many of the algorithms are based, and to the Departments of Statistics at Colorado State University and Statistics and Operations Research at Royal Melbourne Institute of Technology for providing the excellent working environments in which the work was carried out.

Melbourne, Victoria
Fort Collins, Colorado
June, 1996

P.J. Brockwell
R.A. Davis

Contents

Preface	vii
1. Introduction	1
1.1. Examples of Time Series	1
1.2. Objectives of Time Series Analysis	5
1.3. Some Simple Time Series Models	6
1.3.1. Some Zero-Mean Models	7
1.3.2. Models with Trend and Seasonality	9
1.3.3. A General Approach to Time Series Modelling	13
1.4. Stationary Models and the Autocorrelation Function	14
1.4.1. The Sample Autocorrelation Function	17
1.4.2. A Model for the Lake Huron Data	20
1.5. Estimation and Elimination of Trend and Seasonal Components	22
1.5.1. Estimation and Elimination of Trend in the Absence of Seasonality	23
1.5.2. Estimation and Elimination of Both Trend and Seasonality	30
1.6. Testing the Estimated Noise Sequence Problems	34
2. Stationary Processes	43
2.1. Basic Properties	43
2.2. Linear Processes	49
2.3. Introduction to ARMA Processes	53
2.4. Properties of the Sample Mean and Autocorrelation Function	56
2.4.1. Estimation of μ	56
2.4.2. Estimation of $\gamma(\cdot)$ and $\rho(\cdot)$	57
2.5. Forecasting Stationary Time Series	62
2.5.1. The Durbin-Levinson Algorithm	67
2.5.2. The Innovations Algorithm	70
2.5.3. Prediction of a Stationary Process in Terms of Infinitely Many Past Values	73

2.6. The Wold Decomposition	75
Problems	77
3. ARMA Models	81
3.1. ARMA(p, q) Processes	81
3.2. The ACF and PACF of an ARMA(p, q) Process	86
3.2.1. Calculation of the ACVF	86
3.2.2. The Autocorrelation Function	92
3.2.3. The Partial Autocorrelation Function	92
3.2.4. Examples	94
3.3. Forecasting ARMA Processes	98
Problems	106
4. Spectral Analysis	109
4.1. Spectral Densities	110
4.2. The Periodogram	120
4.3. Time-Invariant Linear Filters	126
4.4. The Spectral Density of an ARMA Process	130
Problems	132
5. Modelling and Forecasting with ARMA Processes	135
5.1. Preliminary Estimation	136
5.1.1. Yule-Walker Estimation	137
5.1.2. Burg's Algorithm	145
5.1.3. The Innovations Algorithm	148
5.1.4. The Hannan-Rissanen Algorithm	154
5.2. Maximum Likelihood Estimation	156
5.3. Diagnostic Checking	162
5.3.1. The Graph of $\{\hat{R}_t, t = 1, \dots, n\}$	162
5.3.2. The Sample ACF of the Residuals	163
5.3.3. Tests for Randomness of the Residuals	164
5.4. Forecasting	165
5.5. Order Selection	167
5.5.1. The FPE Criterion	167
5.5.2. The AICC Criterion	169
Problems	172
6. Nonstationary and Seasonal Time Series Models	177
6.1. ARIMA Models for Nonstationary Time Series	178
6.2. Identification Techniques	186

6.3. Unit Roots in Time Series Models	192
6.3.1. Unit Roots in Autoregressions	192
6.3.2. Unit Roots in Moving Averages	195
6.4. Forecasting ARIMA Models	197
6.4.1. The Forecast Function	199
6.5. Seasonal ARIMA Models	201
6.5.1. Forecasting SARIMA Processes	206
6.6. Regression with ARMA Errors	208
Problems	213
7. Multivariate Time Series	217
7.1. Examples	218
7.2. Second-Order Properties of Multivariate Time Series	223
7.3. Estimation of the Mean and Covariance Function	227
7.3.1. Estimation of μ	227
7.3.2. Estimation of $\Gamma(h)$	229
7.3.3. Testing for Independence of Two Stationary Time Series	230
7.3.4. Bartlett's Formula	232
7.4. Multivariate ARMA Processes	234
7.4.1. The Covariance Matrix Function of a Causal ARMA Process	237
7.5. Best Linear Predictors of Second-Order Random Vectors	237
7.6. Modelling and Forecasting with Multivariate AR Processes	239
7.6.1. Estimation for Autoregressive Processes Using Whittle's Algorithm	240
7.6.2. Forecasting Multivariate Autoregressive Processes	242
7.7. Cointegration	247
Problems	248
8. State-Space Models	251
8.1. State-Space Representations	252
8.2. The Basic Structural Model	255
8.3. State-Space Representation of ARIMA Models	259
8.4. The Kalman Recursions	263
8.5. Estimation For State-Space Models	269
8.6. State-Space Models with Missing Observations	275
8.7. The EM Algorithm	281
8.8. Generalized State-Space Models	284
8.8.1. Parameter-Driven Models	284
8.8.2. Observation-Driven Models	291
Problems	303

9. Forecasting Techniques	309
9.1. The ARAR Algorithm	310
9.1.1. Memory Shortening	310
9.1.2. Fitting a Subset Autoregression	311
9.1.3. Forecasting	312
9.1.4. Running the Program ARAR	313
9.2. The Holt-Winters Algorithm	315
9.3. The Holt-Winters Seasonal Algorithm	318
9.4. Choosing a Forecasting Algorithm	320
Problems	322
 10. Further Topics	 323
10.1. Transfer Function Models	323
10.1.1. Prediction Based on a Transfer-Function Model	328
10.2. Intervention Analysis	332
10.3. Nonlinear Models	336
10.3.1. Deviations from Linearity	336
10.3.2. Chaotic Deterministic Sequences	337
10.3.3. Distinguishing Between White Noise and IID Sequences	338
10.3.4. Three Useful Classes of Nonlinear Models	340
10.3.5. Modelling Volatility	341
10.4. Continuous-Time Models	344
10.5. Long-Memory Models	348
Problems	352
 A. Random Variables and Probability Distributions	 355
A.1. Distribution Functions and Expectation	355
A.2. Random Vectors	360
A.3. The Multivariate Normal Distribution	363
Problems	366
 B. Statistical Complements	 369
B.1. Least Squares Estimation	369
B.1.1. The Gauss-Markov Theorem	371
B.1.2. Generalized Least Squares	371
B.2. Maximum Likelihood Estimation	372
B.2.1. Properties of Maximum Likelihood Estimators	373
B.3. Confidence Intervals	373
B.3.1. Large-Sample Confidence Regions	374
B.4. Hypothesis Testing	375

B.4.1. Error Probabilities	375
B.4.2. Large-Sample Tests Based on Confidence Regions	376
C. Mean Square Convergence	379
C.1. The Cauchy Criterion	379
D. An ITSM Tutorial	381
D.1. Getting Started	381
D.1.1. Running PEST	381
D.2. Preparing Your Data for Modelling	382
D.2.1. Entering Data	383
D.2.2. Filing Data	383
D.2.3. Plotting Data	383
D.2.4. Transforming Data	384
D.3. Finding a Model for Your Data	388
D.3.1. The Sample ACF and PACF	388
D.3.2. Entering a Model	389
D.3.3. Preliminary Estimation	391
D.3.4. The AICC Statistic	393
D.3.5. Changing Your Model	394
D.3.6. Maximum Likelihood Estimation	394
D.3.7. Optimization Results	395
D.4. Testing Your Model	396
D.4.1. Plotting the Residuals	397
D.4.2. ACF/PACF of the Residuals	397
D.4.3. Testing for Randomness of the Residuals	399
D.5. Prediction	400
D.5.1. Forecast Criteria	400
D.5.2. Forecast Results	400
D.5.3. Inverting Transformations	401
D.6. Model Properties	402
D.6.1. ARMA Models	403
D.6.2. Model ACF, PACF	403
D.6.3. Model Representations	405
D.6.4. Generating Realizations of a Random Series	406
D.6.5. Spectral Properties	407
Bibliography	409
Index	415