SHADOWS AND SILHOUETTES IN COMPUTER VISION

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Takeo Kanade

SHADOWS AND SILHOUETTES IN COMPUTER VISION

by

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To the memory of Dr. Joseph Shafer, my grandfather.

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