Problem Books in Mathematics

Edited by P. R. Halmos

Problem Books in Mathematics

Series Editor: P.R. Halmos

Unsolved Problems in Intuitive Mathematics, Volume I:

Unsolved Problems in Number Theory

by *Richard K. Guy*

1981. xviii, 161 pages. 17 illus.

Theorems and Problems in Functional Analysis

by *A.A. Kirillov* and *A.D. Gvishiani* (trans. *Harold H. McFaden*)

1982. ix, 347 pages. 6 illus.

Problems in Analysis

by *Bernard Gelbaum*

1982. vii, 228 pages. 9 illus.

A Problem Seminar

by *Donald J. Newman*

1982. viii, 113 pages.

Problem-Solving Through Problems

by *Loren C. Larson*

1983. xi, 344 pages. 104 illus.

Demography Through Problems

by *N. Keyfitz* and *J.A. Beekman*

1984. viii, 141 pages. 22 illus.

Nathan Keyfitz
John A. Beekman

Demography Through Problems

With 22 Illustrations

Springer Science+Business Media, LLC

Nathan Keyfitz
Center for Population Studies
9 Bow Street
Harvard University
Cambridge, MA 02138
U.S.A.

John A. Beekman
Department of Mathematical Sciences
Ball State University
Muncie, IN 47306
U.S.A.

Editor

Paul R. Halmos
Department of Mathematics
Indiana University
Bloomington, IN 47405
U.S.A.

AMS Subject Classifications: 00A07, 60J70, 60J80, 62P10, 92A15

Library of Congress Cataloging in Publication Data

Keyfitz, Nathan, 1913–

Demography through problems.

(Problem books in mathematics)

Bibliography: p.

Includes index.

1. Demography—Problems, exercises, etc. I. Beekman,

John A. II. Title. III. Series.

HB850.3.K49 1984 304.6'076 83-6775

© 1984 by Springer Science+Business Media New York

Originally published by Springer-Verlag Berlin Heidelberg New York in 1984

Softcover reprint of the hardcover 1st edition 1984

All rights reserved. No part of this book may be translated or reproduced in any form without written permission from Springer Science+Business Media, LLC.

Typeset by Composition House Ltd., Salisbury, England.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4419-2817-7 ISBN 978-1-4757-1843-0 (eBook)
DOI 10.1007/978-1-4757-1843-0

Preface

The book that follows is an experiment in the teaching of population theory and analysis. A sequence of problems where each is a self-contained puzzle, and the successful solution of each which puts the student in a position to tackle the next, is a means of securing the active participation of the learner and so the mastery of a technical subject. How far our questions are the exciting puzzles at which we aimed, and how far the sequence constitutes a rounded course in demography, must be left to the user to judge. One test of a good problem is whether a solution, that may take hours of cogitation, is immediately recognizable once it comes to mind.

While algebraic manipulation is required throughout, we have tried to emphasize problems in which there is some substantive point—a conclusion regarding population that can be put into words. Our title, *Demography Through Problems*, reflects our intention of leading the reader who will actively commit him- or herself through a sequence that will not only teach definitions—in itself a trivial matter—but sharpen intuition on the way that populations behave.

We experienced tension between the aim of making the problems demographically relevant, on the one hand, and expressing them in succinct and self-contained formulations, on the other. In demographic practice the complexities of data and their errors, not to mention the sheer volume of material, do not readily produce incisive formulations. A book of this kind is bound to show compromises between incisiveness and the detailed minutiae of real-life data.

Actuaries as well as demographers must utilize national, state, and regional mortality statistics, cause of death statistics, and life tables, and some of the problems are designed to provide experience with the wealth of data from the U.S. Bureau of the Census and other official sources. Students

can pick a Standard Metropolitan Statistical Area (SMSA) and compute its crude death rate, central death rates by age group and sex, and death rates for various causes. As one attempt to explain the revealed differences, students can compute adjusted death rates and age–sex-adjusted death rates for their SMSA's, where the standard population figures were for the state or possibly states surrounding the SMSA's. Each student can build a life table for his or her SMSA. The study of population dynamics must be paralleled by analysis of actual data.

The mere phrasing of problems so that they are clear is a distant goal, hardly to be attained on the first round. Repeated attempts at solution by colleagues and students, and innumerable justified complaints of obscurity, are milestones that have to be passed on the way to clarity. Testing has been possible for many but not all of the problems in this collection. Suggestions for increased precision in the asking of the questions, as well as for rigor in the answers, will be received with gratitude.

The great majority of the questions were devised by ourselves, as well as all of the answers. But our debts are many. Generations of students turned up ambiguities of wording in their attempts to answer the questions. We single out for special mention Michael Stoto, Noreen Goldman, Ronald D. Lee, Juan Carlos Lerda, Robert Lundy, David P. Smith, Edmund M. Murphy, Peter Ellison, Jay Palmore, and John Hsieh; others at Chicago, Berkeley, and Harvard participated. The mathematical demography class of 1981–82 at Ball State University worked through many of the chapters. Parts of the work on the life table in Chapter II were inspired by the examinations of the Society of Actuaries. Thomas Wonnacott (University of Western Ontario) used parts of the manuscript in a class, and provided improvements to several questions. Questions that bear on forecasting developed out of a National Science Foundation grant, whose purpose was to improve forecasting techniques. Ball State University provided a sabbatical to John Beekman, in the course of which he made the major part of his contribution to this joint work. Nathan Keyfitz was greatly helped by the facilities of the Harvard Center for Population Studies and the Sociology Department of Ohio State University.

A number of the questions—though none of the answers—were first published in the revised version of the *Introduction to the Mathematics of Population*, and we are grateful to Addison-Wesley Publishing Company, Reading, Massachusetts, for permission to use these. That book is a source of theory for the reader who wants to go further into the mathematics. A source of a different kind, that emphasizes applications, is *Applied Mathematical Demography* (John Wiley & Sons, 1977), in which many of the ideas that are here expressed as problems will be found.

Finally, the typing of successive drafts of the manuscript was the work of Beatrice Keyfitz and Diane Sorlie.

Contents

CHAPTER I	
Populations That Are Not Age-Dependent	1
1.1 Mathematical Concepts	1
Population Models. Rates of Growth. Exponential and Logistic Models. Matrices and Graphs in Demography.	
1.2. Problems	5
Exercises Based on Mathematical Concepts, U.S. and World Population Figures.	
1.3. Solutions	12
CHAPTER II	
The Life Table	26
2.1. Mathematical Concepts	26
Basic Functions and Their Relations. Force of Mortality. Models for Numbers of Lives. Adjusted Death Rates. Stationary Population Models.	
2.2. Problems	31
Central Death Rate Problems. De Moivre, Uniform, and Balducci Distributions of Deaths. Constant Force of Mortality. Elimination of a Cause of Death. Exercises on Life Table Functions. Effects of Mortality Reductions. Force of Mortality and Expectation of Life. Laws of Gompertz, Makeham, and Weibull.	
2.3. Solutions	40
CHAPTER III	
Uses of Stable Theory	55
Stable Age Distribution. Effects of Birth and Death Rates. Uses of Moments of the Age Distribution. Characteristic Equation in Unknown r .	

3.1. Problems	56
Stable Age Distribution Questions. Filling Gaps in Data. Departure of the Assumptions. Analysis of Age Distributions. Periods and Cohorts.	
3.2. Solutions	61
CHAPTER IV	
Births and Deaths Under Stability	78
4.1. Problems	78
Approximate Solution to Characteristic Equation. Length of Generation. Composite of Stable Subpopulations.	
4.2. Solutions	81
CHAPTER V	
Projection and Forecasting	93
5.1. Problems	93
Extrapolation of Population Totals. Matrix Technique. Analysis of the Projection Matrix. Breakdown by Sex and Other Attributes.	
5.2. Solutions	97
CHAPTER VI	
Stochastic Population Models	110
6.1. Needed Concepts	110
Probabilistic Framework. Random Variables. Mean Value and Variance. Discrete and Continuous Distributions. Generating Functions. Independence. Sums of Random Variables. Central Limit Theorem.	
6.2. Problems	115
Family Size Exercises. Probability of Extinction. Expected Number of Direct Descendents. Distribution of Sizes of Completed Families. Parity Progression Ratio. Rural–Urban Shifts and Markov Chains. Ergodic Theorems for Markov Chains and Demography. Birth and Death Stochastic Processes. Population Estimates and Loss Functions.	
6.3. Solutions	124
Brief List of References	135
Index	137