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Kai Lai Chung

Lectures from Markov Processes to Brownian Motion

With 3 Figures



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Kai Lai Chung
Department of Mathematics
Stanford University
Stanford, CA 94305

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Preface

This book evolved from several stacks of lecture notes written over a decade and given in classes at slightly varying levels. In transforming the overlapping material into a book, I aimed at presenting some of the best features of the subject with a minimum of prerequisites and technicalities. (Needless to say, one man's technicality is another's professionalism.) But a text frozen in print does not allow for the latitude of the classroom; and the tendency to expand becomes harder to curb without the constraints of time and audience. The result is that this volume contains more topics and details than I had intended, but I hope the forest is still visible with the trees.

The book begins at the beginning with the Markov property, followed quickly by the introduction of optional times and martingales. These three topics in the discrete parameter setting are fully discussed in my book *A Course In Probability Theory* (second edition, Academic Press, 1974). The latter will be referred to throughout this book as the *Course*, and may be considered as a general background; its specific use is limited to the material on discrete parameter martingale theory cited in §1.4. Apart from this and some dispensable references to Markov chains as examples, the book is self-contained. However, there are a very few results which are explained and used, but not proved here, the first instance being the theorem on projection in §1.6. The fundamental regularity properties of a Markov process having a Feller transition semigroup are established in Chapter 2, together with certain measurability questions which must be faced. Chapter 3 contains the basic theory as formulated by Hunt, including some special topics in the last three sections. Elements of a potential theory accompany the development, but a proper treatment would require the setting up of dual structures. Instead, the relevant circle of ideas is given a new departure in Chapter 5. Chapter 4 grew out of a short compendium as a particularly telling example, and Chapter 5 is a splinter from unincorporated sections of Chapter 4. The venerable theory of Brownian motion is so well embellished and ramified that once begun it is hard to know where to stop. In the end I have let my own propensity and capability make the choice. Thus the last three sections of the book treat several recent developments which have engaged me lately. They are included here with the hope of inducing further work in such fascinating old-and-new themes as equilibrium, energy, and reversibility.

I used both the Notes and Exercises as proper non-trivial extensions of the text. In the Notes a number of regrettably omitted topics are mentioned, and related to the text as a sort of guide to supplementary reading. In the Exercises there are many alternative proofs, important corollaries and examples that the reader will do well not to overlook.

The manuscript was prepared over a span of time apparently too long for me to maintain a uniform style and consistent notation. For instance, who knows whether “semipolar” should be spelled with or without a hyphen? And if both $|x|$ and $\|x\|$ are used to denote the same thing, does it really matter? Certain casual remarks and repetitions are also left in place, as they are permissible, indeed desirable, in lectures. Despite considerable pains on the part of several readers, it is perhaps too much to hope that no blunders remain undetected, especially among the exercises. I have often made a point, when assigning homework problems in class, to say that the correction of any inaccurate statement should be regarded as part of the exercise. This is of course not a defense for mistakes but merely offered as prior consolation.

Many people helped me with the task. To begin with, my first formal set of notes, contained in five folio-size, lined, students' copybooks, was prepared for a semester course given at the Eidgenössische Technische Hochschule in the spring of 1970. My family has kept fond memories of a pleasant sojourn in a Swiss house in the great city of Zürich, and I should like to take this belated occasion to thank our hospitable hosts. Another set of notes (including the lectures given by Doob mentioned in §4.5) was taken during 1971–2 by Harry Guess, who was kind enough to send me a copy. Wu Rong, a visiting scholar from China, read the draft and the galley proofs, and checked out many exercises. The comments by R. Getoor, N. Falkner, and Liao Ming led to some final alterations. Most of the manuscript was typed by Mrs. Gail Stein, who also typed some of my other books. Mrs. Charlotte Crabtree, Mrs. Priscilla Feigen, and my daughter Marilda did some of the revisions. I am grateful to the National Science Foundation for its support of my research, some of which went into this book.

August 1981

Kai Lai Chung