Graduate Texts in Mathematics 12

Richard Beals

Advanced Mathematical Analysis

Periodic Functions and Distributions, Complex Analysis, Laplace Transform and Applications

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to Nancy

PREFACE

Once upon a time students of mathematics and students of science or engineering took the same courses in mathematical analysis beyond calculus. Now it is common to separate "advanced mathematics for science and engineering" from what might be called "advanced mathematical analysis for mathematicians." It seems to me both useful and timely to attempt a reconciliation.

The separation between kinds of courses has unhealthy effects. Mathematics students reverse the historical development of analysis, learning the unifying abstractions first and the examples later (if ever). Science students learn the examples as taught generations ago, missing modern insights. A choice between encountering Fourier series as a minor instance of the representation theory of Banach algebras, and encountering Fourier series in isolation and developed in an *ad hoc* manner, is no choice at all.

It is easy to recognize these problems, but less easy to counter the legitimate pressures which have led to a separation. Modern mathematics has broadened our perspectives by abstraction and bold generalization, while developing techniques which can treat classical theories in a definitive way. On the other hand, the applier of mathematics has continued to need a variety of definite tools and has not had the time to acquire the broadest and most definitive grasp—to learn necessary and sufficient conditions when simple sufficient conditions will serve, or to learn the general framework encompassing different examples.

This book is based on two premises. First, the ideas and methods of the theory of distributions lead to formulations of classical theories which are satisfying and complete mathematically, and which at the same time provide the most useful viewpoint for applications. Second, mathematics and science students alike can profit from an approach which treats the particular in a careful, complete, and modern way, and which treats the general as obtained by abstraction for the purpose of illuminating the basic structure exemplified in the particular. As an example, the basic L^2 theory of Fourier series can be established quickly and with no mention of measure theory once $L^2(0, 2\pi)$ is known to be complete. Here $L^2(0, 2\pi)$ is viewed as a subspace of the space of periodic distributions and is shown to be a Hilbert space. This leads to a discussion of abstract Hilbert space and orthogonal expansions. It is easy to derive necessary and sufficient conditions that a formal trigonometric series be the Fourier series of a distribution, an L^2 distribution, or a smooth function. This in turn facilitates a discussion of smooth solutions and distribution solutions of the wave and heat equations.

The book is organized as follows. The first two chapters provide background material which many readers may profitably skim or skip. Chapters 3, 4, and 5 treat periodic functions and distributions, Fourier series, and applications. Included are convolution and approximation (including the

Weierstrass theorems), characterization of periodic distributions, elements of Hilbert space theory, and the classical problems of mathematical physics. The basic theory of functions of a complex variable is taken up in Chapter 6. Chapter 7 treats the Laplace transform from a distribution-theoretic point of view and includes applications to ordinary differential equations. Chapters 6 and 7 are virtually independent of the preceding three chapters; a quick reading of sections 2, 3, and 5 of Chapter 3 may help motivate the procedure of Chapter 7.

I am indebted to Max Jodeit and Paul Sally for lively discussions of what and how analysts should learn, to Nancy for her support throughout, and particularly to Fred Flowers for his excellent handling of the manuscript.

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