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I. J. Bienaymé
Statistical Theory Anticipated

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. . . mais il faut parier. Cela n'est pas volontaire: vous êtes embarqué.

Blaise Pascal, *Pensées*

Preface

Our interest in I. J. Bienaymé was kindled by the discovery of his paper of 1845 on simple branching processes as a model for extinction of family names. In this work he announced the key criticality theorem 28 years before it was rediscovered in incomplete form by Galton and Watson (after whom the process was subsequently and erroneously named). Bienaymé was not an obscure figure in his time and he achieved a position of some eminence both as a civil servant and as an Academician. However, his name is no longer widely known. There has been some recognition of his work on least squares, and a gradually fading attribution in connection with the (Bienaymé–) Chebyshev inequality, but little more. In fact, he made substantial contributions to most of the significant problems of probability and statistics which were of contemporary interest, and interacted with the major figures of the period. We have, over a period of years, collected his traceable scientific work and many interesting features have come to light. The present monograph has resulted from an attempt to describe his work in its historical context. Earlier progress reports have appeared in Heyde and Seneta (1972, to be reprinted in *Studies in the History of Probability and Statistics*, Volume 2, Griffin, London; 1975; 1976).

It is our aim in this monograph to focus on Bienaymé's work in its context, both for its intrinsic interest and for the perspective it gives on developments in the 19th century. Indeed, the evolution of probability and statistics is fairly well documented up to the time of Laplace, and the developments of the twentieth century are widely appreciated. The intervening period of the last three quarters of the nineteenth century is the least well-understood period in the history of the subject. Reasons for this are not hard to find. The technical nature of the work of the period requires a substantial grounding in the subject on the part of its interpreters, as well as due caution in the imposition of modern interpretations. Furthermore, sources are frequently obscure enough to provide significant risks of the overlooking of important material and, at the same time, hindering early completion of most investigations. A work such as ours must inevitably be flawed by errors of omission and misinterpretation; of course we have tried to minimize them.

The timing of this book is opportune, since 1978 is the centenary of Bienaymé's death. In 1974 the centenary of the death of his close contem-

porary Quetelet was celebrated with considerable fanfare. A commemorative session was organized by the Royal Academy of Belgium in December 1974, and Belgium issued a commemorative portrait stamp at the same time. In addition, a group meeting at the 40th ISI Meeting in September 1975 was devoted to Quetelet. Centennial papers commemorating the contribution of Galton and Watson to branching processes (predated by that of Bienaymé) appeared in the journal *Advances in Applied Probability* in 1974.

There are various structural points concerning the monograph which should be remarked upon. First, quotations originally in French, German, and Russian are given in free English translation which seeks to capture the spirit rather than the letter of the original.

Next, there are several tables. Table 1 (in the front matter) lists the chronology of the most relevant scientists mentioned in the text; and Table 2 (at the back of the book) the references to Bienaymé extracted from the name indexes of *Comptes Rendus Hebd. des Séances de l'Académie des Sciences*. Bienaymé's publications are listed separately, rather than as part of the general bibliography.

In the case of a few individuals and books, we have not been able to positively ascertain full details on items such as initials, place of publication, or publisher. Also, a number of papers and books mentioned have been reprinted. In these cases we have sought to give at least one bibliographically complete reference, while mentioning others.

Finally, in cases where not only multiple, but confusing, versions of names exist, generally due to transliteration, we have used the following: Bortkiewicz, Chebyshev, Chuprov, Hanikov, Iastremsky, Liapounov, Sleshinsky, even though these authors may have themselves used different transliterations (which are indicated in the reference list).

The extant memorabilia of I. J. Bienaymé which we have been able to trace consist of the following:

(i) A dossier on I. J. Bienaymé in the Archive of the Académie des Sciences, Paris, which contains, *inter alia*, a photograph of I. J. B. (reproduced as our frontispiece), a note concerning the location of his tomb at the Montparnasse cemetery, a decree concerning his election to the Academy, and certain documents pertaining to his death and that of his wife (which are discussed in our Chapter 1).

(ii) Three letters from I. J. B. to Chebyshev, in the Archive of the Academy of Sciences of the U.S.S.R. These are reprinted in Russian translation in Chebyshev (1951) and are discussed in our Chapter 1.

(iii) A photograph of M. (= Monsieur) Bienaymé preserved in the *Département des Estampes de la Bibliothèque Nationale*, but without initials or date.

Finally, acknowledgments are due, first, to O. B. Sheynin for invaluable assistance with materials; and also to P. Aukland, C. Eisenhart, R. A. Horváth, P. Jagers, H. O. Lancaster, D. R. McNeil, H. Seal, H. Solomon,

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C. C. Heyde
E. Seneta

Canberra, A.C.T. and
Blacksburg, VA
March, 1977

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Table 1. Chronology of most relevant
scientists mentioned in the text
(by year of birth and nationality)

French

A. Deparcieux	1703–1768
“Moheau”*	1733–1820
J. A. N. de Caritat de Condorcet	1743–1794
P. S. de Laplace	1749–1827
A. Legendre	1752–1833
E. E. Duvillard	1755–1832
J. Fourier	1768–1830
L. F. Benoiston de Châteauneuf	1776–1856
S. D. Poisson	1781–1840
D. F. Arago	1786–1853
A. L. Cauchy	1789–1857
G. Lamé	1795–1870
I. J. Bienaymé	1796–1878
A. A. Cournot	1801–1877
J. Liouville	1809–1882
C. Hermite	1822–1901
J. L. F. Bertrand	1842–1912

Other

R. Price	1723–1791
T. R. Malthus	1766–1834
T. Doubleday	1790–1870
F. Galton	1822–1911
H. W. Watson	1827–1903

*Pen name of J. B. A. R. Auget, Baron de Montyon.

C. F. Gauss	1777–1855
W. Lexis	1837–1914
L. A. J. Quetelet	1796–1874
M. V. Ostrogradsky	1801–1862
V. A. Buniakovsky	1804–1889
P. L. Chebyshev	1821–1894
A. A. Markov	1856–1922
L. J. Bortkiewicz	1868–1931
A. A. Chuprov	1874–1926
A. P. de Candolle	1806–1893