Undergraduate Texts in Mathematics

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Calculus: An Historical Approach

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To William Montgomery
and Thomas Carter,
for their delight in
sandpiles, seesaws, and pebbles.

Even now there is a very wavering grasp of the true position of mathematics as an element in the history of thought. I will not go so far as to say that to construct a history of thought without profound study of the mathematical ideas of successive epochs is like omitting Hamlet from the play which is named after him. That would be claiming too much. But it is certainly analogous to cutting out the part of Ophelia. This simile is singularly exact. For Ophelia is quite essential to the play, she is very charming—and a little mad. Let us grant that the pursuit of mathematics is a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings.

Alfred North Whitehead
from *Mathematics as an Element in the History of Thought*

Preface

This book is for students being introduced to calculus, and it covers the usual topics, but its spirit is different from what might be expected. Though the approach is basically historical in nature, emphasis is put upon ideas and their place—not upon events and their dates. Its purpose is to have students to learn calculus first, and to learn incidentally something about the nature of mathematics.

Somewhat to the surprise of its author, the book soon became animated by a spirit of opposition to the darkness that separates the sciences from the humanities. To fight the spell of that darkness anything at hand is used, even a few low tricks or bad jokes that seemed to offer a slight promise of success. To lighten the darkness, to illuminate some of the common ground shared by the two cultures, is a goal that justifies almost any means. It is possible that this approach may make calculus more fun as well.

Whereas the close ties of mathematics to the sciences are well known, the ties binding mathematics to the humanities are rarely noticed. The result is a distorted view of mathematics, placing it outside the mainstream of liberal arts studies. This book tries to suggest gently, from time to time, where a kinship between mathematics and the humanities may be found.

There is a misconception today that mathematics has mainly to do with scientific technology or with computers, and is thereby unrelated to humanistic thought. One sees textbooks with such titles as *Mathematics for Liberal Arts Majors*, a curious phrase that seems to suggest that the liberal arts no longer include mathematics.

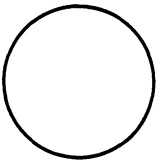
No discipline has been a part of liberal arts longer than mathematics. Three—logic, arithmetic, and geometry—of the original seven liberal arts are branches of mathematics. Plato's friend Archytas, who helped develop the whole idea of liberal education, was a distinguished mathematician. No true student of liberal arts can neglect mathematics.

How did it happen that mathematics, in the public eye, became dissociated from the humanities? In brief, the emergence and growth of scientific knowledge in the seventeenth century led to a polarization in academic circles. Science went one way, the humanities went another. Mathematics, at first in the middle, seems now to be more commonly identified with the sciences and with the technology they engendered.

Today in some academic institutions the state is not healthy. The ground between the sciences and the humanities is so dark that many well-meaning members on each side lack the education to see the most valuable contributions of the other. To the disadvantage of students, this is sometimes the case even among the faculties of so-called "liberal arts" colleges.

In the seventeenth century mathematics was a bridge between the two kinds of knowledge. Thus, for example, Isaac Newton's new physics could be read by Voltaire, who was at home both with Homer and with Archimedes. Voltaire even judged Archimedes to be superior, in imagination, to Homer.

The unity of knowledge which seemed attainable in the seventeenth century, and which has long been an ideal of liberal education, is still worth seeking. Today as in the time of Voltaire, and in the time of Plato, mathematics calls us to eye this goal.



For Anyone Afraid of Mathematics

Maturity, it has been said, involves knowing when and how to delay succumbing to an urge, in order by doing so to attain a deeper satisfaction. To be immature is to demand, like a baby, the immediate gratification of every impulse.

Perhaps happily, none of us is mature in every respect. Mature readers of poetry may be immature readers of mathematics. Statemen mature in diplomacy may act immaturely in dealing with their own children. And mature mathematicians may on occasion act like babies when asked to listen to serious music, to study serious art, or to read serious poetry.

What is involved in many such cases is how we control our natural urge to get directly to the point. In mathematics, as in serious music or literature, the point sometimes simply cannot be attained immediately, but only by indirection or digression.

The major prerequisite for reading this book is a willingness to cultivate some measure of maturity in mathematics. If you get stuck, be willing to forge ahead, with suspended disbelief, to see where the road is leading. "Go forward, and faith will follow!" was d'Alembert's advice in the eighteenth century to those who would learn the calculus. Your puzzlement may vanish upon turning a page.

All that will be assumed at the outset is a nodding acquaintance with some elementary parts of arithmetic, algebra, and geometry, most of which was developed long before A.D. 1600. There will be some review in the early chapters, offering us as well a chance to outline the early history of mathematics.

To the Instructor

This book aspires to aid a student interested in either

(1) receiving an elementary introduction to the basic ideas of calculus; or
(2) learning "about" calculus, as a significant element in the history of thought.

At first these goals may seem incompatible. In fact, each tends to reinforce the other. A remark made by George Pólya suggests how both goals might be accomplished at once.

> If the learning of mathematics reflects to any degree the invention of mathematics, it must have a place for guessing, for plausible inference.

The reader will find plenty of opportunity here for guessing. The early chapters go at a gentle pace and invite the reader to enter into the spirit of the investigation.

For those whose backgrounds in mathematics are not especially strong, Chapters 1–6, together with Chapter 10, have been designed to form *a terminal course in mathematics.* (Chapter 10 can be read immediately after Chapter 6.) In these chapters algebraic manipulations have been kept at a simple level. Negative and fractional exponents can be avoided altogether here, and even the absolute value function is omitted. Trigonometry is not introduced until Chapter 7. The result, even with the omission of some of the harder problems at the ends of chapters, is not a watered-down course, but one that retains the full flavor of calculus.

This book is intended for use in a different way by students well prepared in mathematics. They should move rapidly through Chapters 1–6, omitting many of the routine exercises placed at the ends of sections and concentrating

upon the more challenging problem sets located at the ends of chapters. After Chapter 6 the pace of the book picks up moderately. Chapter 7 develops the calculus of trigonometric functions, following a quick introduction to trigonometry. Chapter 8 discusses the integral in more detail than before, adding a little more rigor to a treatment that remains basically intuitive. Chapter 9 deals with the exponential and logarithmic functions. Chapter 10 is mainly historical in nature, but it sets the stage for the study of integration techniques through the use of formal manipulations with differentials.

The book ends with Chapter 10, where a leisurely two-semester calculus course will find itself near the end of the year. In a fast-moving course the instructor may have to look elsewhere for additional material to cover. Had there been a Chapter 11, it would have covered techniques of integration, a subject often touched upon in Chapters 6–10 but whose systematic study was postponed. The theory of infinite series is another important topic that has been touched upon but not developed in this volume.

In time, perhaps, a second volume will appear—to include these and other topics and to bring the story more up-to-date.

I wish to thank Mary Priestley for helping me in this enterprise and for sharing with me its ups and downs. I am grateful also to Paul Halmos for his interest and encouragement.

W. M. P.

May, 1978

Sewanee

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Anecdote of the Jar

Wallace Stevens

I placed a jar in Tennessee
And round it was, upon a hill.
It made the slovenly wilderness
Surround that hill.

The wilderness rose up to it,
And sprawled around, no longer wild.
The jar was round upon the ground
And tall and of a port in air.

It took dominion everywhere.
The jar was gray and bare.
It did not give of bird or bush,
Like nothing else in Tennessee.