Undergraduate Texts in Mathematics

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Elementary Stability and Bifurcation Theory

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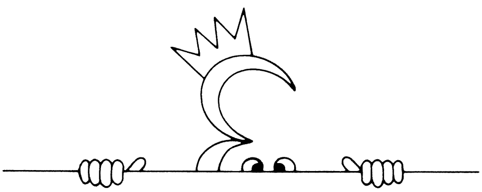
Everything should be made as simple as possible,
but not simpler.

ALBERT EINSTEIN



Contents

List of Frequently Used Symbols

All symbols are fully defined at the place where they are first introduced. As a convenience to the reader we have collected some of the more frequently used symbols in several places. The largest collection is the one given below. Shorter lists, for later use can be found in the introductions to Chapters X and XI.

$\stackrel{\text{def}}{=}$	equality by definition
$\in$	"$a \in A$" means "a belongs to the set A" or "a is an element of A"
$\mathbb{N}$	the set of nonnegative integers (0 included)
$\mathbb{N}^*$	the set of strictly positive integers (0 excluded)
$\mathbb{Z}$	the set of positive and negative integers including 0
$\mathbb{R}$	the set of real numbers (the real line)
$\mathbb{R}^n$	the set of ordered n-tuples of real numbers $\mathbf{a} \in \mathbb{R}^n$ may be represented as $\mathbf{a} = (a_1, \ldots, a_n)$. Moreover, $\mathbb{R}^n$ is a Euclidian space with the scalar product $\langle \mathbf{a}, \mathbf{b} \rangle = \sum_{i=1}^{n} a_i b_i$ where $\mathbf{a} = (a_1, \ldots, a_n), \mathbf{b} = (b_1, \ldots, b_n)$. $\mathbb{R}^1 = \mathbb{R}$; $\mathbb{R}^2$ is the plane
$\mathbb{C}$	the set of complex numbers
$\mathbb{C}^n$	the set of ordered n-tuples of complex numbers. The scalar product in $\mathbb{C}^n$ is denoted as in $\mathbb{R}^n$, but $\langle \mathbf{a}, \mathbf{b} \rangle = \sum_{i=1}^{n} a_i \overline{b_i} = \overline{\langle \mathbf{b}, \mathbf{a} \rangle}$.

$\mathscr{C}^n(\mathscr{V})$ — the set of n-times continuously differentiable functions on a domain $\mathscr{V}$. We may furthermore specify the domain E where these functions take their values by writing $\mathscr{C}^n(\mathscr{V}; E)$.

$\|\mathbf{u}\|$ — the norm of $\mathbf{u}$. For instance, if $\mathbf{u} \in \mathbb{C}^n$ we have $\|\mathbf{u}\| = \langle \mathbf{u}, \mathbf{u} \rangle^{1/2}$; if $\mathbf{u} \in \mathscr{C}(\mathscr{V})$, $\|\mathbf{u}\| = \text{l.u.b}_{x \in \mathscr{V}} \|\mathbf{u}(x)\|$, where $\|\mathbf{u}(x)\|$ is the norm of $\mathbf{u}(x)$ in the domain of values for $\mathbf{u}$; $\|\mathbf{u}\| = 0$ implies that $\mathbf{u} = 0$.

$\mathbf{A}(\cdot)$ — a linear operator:

$$\mathbf{A}(\alpha\mathbf{u} + \beta\mathbf{v}) = \alpha\mathbf{A}(\mathbf{u}) + \beta\mathbf{A}(\mathbf{v}).$$

$\mathbf{B}(\cdot, \cdot)$ — a bilinear operator:

$$\mathbf{B}(\alpha_1\mathbf{u}_1 + \alpha_2\mathbf{u}_2, \beta_1\mathbf{v}_1 + \beta_1\mathbf{v}_2) = \alpha_1\beta_1\mathbf{B}(\mathbf{u}_1, \mathbf{v}_1) + \alpha_1\beta_2\mathbf{B}(\mathbf{u}_1, \mathbf{v}_2) + \alpha_2\beta_1\mathbf{B}(\mathbf{u}_2, \mathbf{v}_1) + \alpha_2\beta_2\mathbf{B}(\mathbf{u}_2, \mathbf{v}_2)$$

$\mathbf{C}(\cdot, \cdot, \cdot)$ — a trilinear operator

$\mathbf{N}(\cdot)$ — a general nonlinear operator with no constant term and no linear term in a neighborhood of 0:

$$\mathbf{N}(\mathbf{u}) \stackrel{\text{def}}{=} B(\mathbf{u}, \mathbf{u}) + C(\mathbf{u}, \mathbf{u}, \mathbf{u}) + O(\|\mathbf{u}\|^4)$$

Sometimes we assign a slightly different meaning to **A**, **B**, **C**:

$$(\mathbf{A} \cdot \mathbf{u})_i = A_{ij}u_j = A_{i1}u_1 + A_{i2}u_2 + \cdots + A_{in}u_n$$

$$(\mathbf{B} \cdot \mathbf{u} \cdot \mathbf{v})_i = B_{ijk}u_jv_k$$

$$(\mathbf{C} \cdot \mathbf{u} \cdot \mathbf{v} \cdot \mathbf{w})_i = C_{ijkl}u_jv_kw_l$$

where we use the summation convention for repeated indices and where

(A_{ij}) is the matrix of a linear operator

(B_{ijk}) is the matrix of a bilinear operator

(C_{ijkl}) is the matrix of a trilinear operator

$\mathbf{F}(t, \mu, \mathbf{U})$ — a nonlinear operator—see the opening paragraph of Chapter I

$\mathbf{f}(t, \mu, \mathbf{u})$ — reduction of **F** to "local form," see §I.3

$\mathbf{F}_u$, $\mathbf{F}_{uu}$, etc. — derivatives of **F**; see §I.6–7

$\mathbf{F}_u(t, \mu, \mathbf{U}_0 | \cdot)$ — the linear operator associated with the derivative of **F** at $\mathbf{U} = \mathbf{U}_0$

$\mathbf{F}_u(t, \mu, \mathbf{U}_0 | \mathbf{v})$ — first derivative of $\mathbf{F}(t, \mu, \mathbf{U})$, evaluated at $\mathbf{U} = \mathbf{U}_0$, acting on $\mathbf{v}$

$\sigma = \xi + i\eta$ — an eigenvalue of a linear operator arising in the study of stability of $\mathbf{u} = 0$

When $\mathbf{u} = 0$ corresponds to a time-periodic $\mathbf{U}(t) = \mathbf{U}(t + T)$, then σ is a *Floquet exponent*

$\lambda = e^{\sigma T}$	a *Floquet multiplier*; see preceding entry and §VII.6.2
$\gamma = \xi + i\eta$	an eigenvalue of a linear operator arising in the study of bifurcating solution. We use the same notation, ξ and η, for the real and imaginary part of σ and γ and depend on the context to define the difference.
ω; $T = 2\pi/\omega$	frequency ω and period T
ε	amplitude of a bifurcating solution defined in various ways: under (II.2), (V.2), (VI.72), $(\text{VII.6})_2$, (VIII.22), Figure X.1.
$\langle \mathbf{a}, \mathbf{b} \rangle$	notation for a scalar product $\overline{\langle \mathbf{a}, \mathbf{b} \rangle} = \langle \mathbf{b}, \mathbf{a} \rangle$ with the usual conventions. For vectors in $\mathbb{C}^n$, $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a} \cdot \bar{\mathbf{b}}$. For vector fields in $\mathscr{V}$, $\langle \mathbf{a}, \mathbf{b} \rangle = \int_{\mathscr{V}} \mathbf{a}(\mathbf{x})\bar{\mathbf{b}}(\mathbf{x})\, d\mathscr{V}$. See (IV.7), under (VI.4), §VI.6, under $(\text{VI.134})_2$, and under (VI.144)
$[\mathbf{a}, \mathbf{b}]$	another scalar product for 2π-periodic functions, defined above (VIII.15)
$[\mathbf{a}, \mathbf{b}]_{nT}$	see (IX.16)

Some operators whose domains are 2π-periodic functions of s:

$$J(\cdot, \varepsilon), \qquad J(\cdot, 0) = J_0 \text{ (VII.38)}$$

$$\mathbb{J}_0 \text{ (VIII.15)}; \qquad \mathbb{J}_0^* \text{ (VIII.19)}; \qquad \mathbb{J}(\varepsilon) \text{ above (VIII.37)}.$$

Similar operators for nT-periodic functions are defined under notation for Chapter IX, at the beginning of Chapter IX.

Order Symbols. we say that $f(\varepsilon) = O(\varepsilon^n)$ if

$$\frac{f(\varepsilon)}{\varepsilon^n} \text{ is bounded when } \varepsilon \to 0$$

we say that $f(\varepsilon) = o(\varepsilon^n)$

$$\lim_{\varepsilon \to 0} \frac{f(\varepsilon)}{\varepsilon^n} = 0.$$