B

Progress in Mathematics
Vol. 23

Edited by
J. Coates and
S. Helgason

Birkhäuser
Boston · Basel · Stuttgart

A. Weil
**Adeles
and Algebraic Groups**

1982

Birkhäuser
Boston • Basel • Stuttgart

Author:

A. Weil
The Institute for Advanced Study
Princeton, New Jersey 08540

Library of Congress Cataloging in Publication Data

Weil, André, 1906-

Adeles and algebraic groups.

(Progress in mathematics ; v. 23)

"Notes are based on lectures, given at the
Institute for Advanced Study in 1959-1960"--
Foreword.

Bibliography: p.

1. Forms, Quadratic. 2. Linear algebraic
groups. 3. Adeles. I. Title. II. Series:
Progress in mathematics (Cambridge, Mass.) ;
v. 23.

QA243.W44 1982 512.9'44 82-12767

CIP - Kurztitelauf der Deutschen Bibliothek

Weil, Andre:

Adeles and algebraic groups / Andre Weil. -

Boston; Basel; Stuttgart : Birkhäuser, 1982.

(Progress in mathematics; 23)

NE: GT

All rights reserved. No part of this publication may be reproduced,
stored in a retrieval system, or transmitted, in any form or by any
means, electronic, mechanical, photocopying, recording or otherwise,
without prior permission of the copyright owner.

© Birkhäuser Boston, 1982

ISBN 978-1-4684-9158-6 ISBN 978-1-4684-9156-2 (eBook)

DOI 10.1007/978-1-4684-9156-2

F O R E W O R D

The present notes are based on lectures, given at the Institute for Advanced Study in 1959-1960, which, in a sense, were nothing but a commentary on various aspects of Siegel's work—chiefly his classical papers on quadratic forms, but also the later papers where the volumes of various fundamental domains are computed.

The very fruitful idea of applying the adèle method to such problems comes from Tamagawa, whose work on this subject is not yet published; I was able to make use of a manuscript of his, where that idea was applied to the restatement and proof of Siegel's theorem on quadratic forms.

If the reader is able to derive some profit from these notes, he will owe it, to a large extent, to M. Demazure and T. Ono, who have greatly improved upon the oral presentation of this material as given in my lectures. At many points they have acted as collaborators rather than as note-takers. If the final product is not as pleasing to the eye as one could wish, this is not their fault; it indicates that much work remains to be done before this very promising topic reaches some degree of completion.

Princeton, January 15, 1961

A. WEIL

TABLE OF CONTENTS

	<u>Page</u>
CHAPTER I. PRELIMINARIES ON ADELE-GEOMETRY	
1.1. Adeles	1
1.2. Adele-spaces attached to algebraic varieties	1
1.3. Restriction of the basic field	4
CHAPTER II. TAMAGAWA MEASURES	
2.1. Preliminaries	10
2.2. The case of an algebraic variety : the local measure	13
2.3. The global measure and the convergence factors	21
2.4. Algebraic groups and Tamagawa numbers	22
CHAPTER III. THE LINEAR, PROJECTIVE AND SYMPLECTIC GROUPS	
3.1. The zeta-function of a central division algebra	30
3.2. The projective group of a central division algebra	41
3.3. Isogenies	43
3.4. End of proof of Theorem 3.3.1. : central simple algebras	47
3.5. The symplectic group	52
3.6. Isogenies for products of linear groups	54
3.7. Application to some orthogonal and hermitian groups	61
3.8. The zeta-function of a central simple algebra	65
CHAPTER IV. THE OTHER CLASSICAL GROUPS	
4.1. Classification and general theorems	72
4.2. End of proof of Theorem 4.1.3 (types O_1 , $L_2(a)$, S_2)	84
4.3. The local zeta-functions for a quadratic form	88
4.4. The Tamagawa number (hermitian and quaternionic cases)	91

4.5. The Tamagawa number of the orthogonal group	100
APPENDIX 1. (by M. Demazure)	
The case of the group G_2	111
APPENDIX 2. (by T. Ono)	
A short survey of subsequent research on Tamagawa numbers	114