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George McCarty

University of California, Irvine

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PREFACE

HOW THIS BOOK DIFFERS

This book is about the calculus. What distinguishes it, however, from other books is that it uses the pocket calculator to illustrate the theory. A computation that requires hours of labor when done by hand with tables is quite inappropriate as an example or exercise in a beginning calculus course. But that same computation can become a delicate illustration of the theory when the student does it in seconds on his calculator. † Furthermore, the student's own personal involvement and easy accomplishment give him reassurance and encouragement.

The machine is like a microscope, and its magnification is a hundred millionfold. We shall be interested in limits, and no stage of numerical approximation proves anything about the limit. However, the derivative of $f(x) = 67.89^x$, for instance, acquires real meaning when a student first appreciates its values as numbers, as limits of

† A quick example is 1.1^{10} , 1.01^{100} , 1.001^{1000} ,

Another example is $t = 0.1, 0.01, \dots$ in the function $(\sqrt{3t+9}-3)/t$.

difference quotients of numbers, rather than as values of a function that is itself the result of abstract manipulation.

Similarly, the fun and excitement a student has in calculating for himself some approximations to a few definite integrals, such as $\int_0^1 \sqrt{1-x^2} dx$, will give reality to their definition as limits of Riemann sums. When our usual algebraic manipulation of the sums for the integrands 1 , x , x^2 , and perhaps x^3 is augmented by such calculations, the Fundamental Theorem of the Calculus is seen in a new light. Instead of being misunderstood to be part of the definition of the integral, it becomes a genuine theorem that usefully relates two disparate mathematical objects.

This is not a manual of machine usage, but the student who works through this book will gain calculational competence and a skill at coaxing the most from his machine. Although it is not a workbook of numerical analysis, this book will introduce that subject--there are discussions and examples of errors, numerical quadrature, finding zeros, evaluating functions, and solving differential equations numerically.

The student learns respect here for calculation in problems where theoretical methods fail and only numerical solutions exist. However, in other problems, after he labors to form a few partial sums for a series like $1 - 1/3 + 1/5 - \dots$, he will appreciate the ease and power with which the theory gives the limiting value. Perhaps now the calculator's buttons and twinkling lights can seduce the student to a balanced understanding of the theory and practice of the calculus.

There has been no attempt to be complete in the exposition of theory in this book, but the most important theorems are cited explicitly and illustrated numerically. The student may use these citations as signals for review in his conventional calculus text. The chapters are short; each one stands as independently as the underlying theory will allow. Discussions and detailed solutions for several *Examples* are included in each chapter. In addition there are both *Exercises* and *Problems*. The Exercises are easy and to the point. Some Exercises include applications drawn from the biological, social, and physical sciences. The Problems are more difficult or longer, often

they explore less central topics, and some ask for proofs. Answers to starred Exercises and Problems are given at the end of each chapter.

CLASSROOM USE IN A FIRST CALCULUS COURSE

When this book is used as a workbook or problem manual for an introductory course that also has a conventional text, the instructor may concentrate on the demonstrative Examples and Exercises. Much of the explanatory material in this book may be left for the student to read as he studies. Roughly half of all assigned exercises might be chosen from this book and half from the conventional text. The author usually devotes one of his three weekly lectures plus one of the two weekly problem sessions to material involving the calculator.

The first two chapters in this book include topics outside the usual preliminary material for the calculus. The algorithms for square roots and for successive substitutions serve to introduce functions and graphs. They also accustom students to their machines and start them thinking about limiting processes. But Chapter 1 may be omitted entirely, and Chapter 2 may be omitted if the method of successive substitutions is briefly explained when it is needed in Exercises in Chapters 6 and 8.

CLASSROOM USE FOR OTHER COURSES

This book can serve as the text for an advanced undergraduate course. For such a course it will be appropriate to consider many of the more difficult Problems. It can also be used for a one-semester or one-quarter course having first-year calculus as prerequisite or co-requisite. At this lower level, most students will find enough challenge in the Exercises, with few Problems attempted.

The author feels it is important to plan the schedule of such a course so that the material on series, in Chapters 11, 12, and 13, is sure to be covered. If necessary, Chapters 9 or 10 or both, on volumes and on curves and polar coordinates, may be omitted with no loss of continuity. Chapter 14 may also be omitted in a lower-level course, particularly if students have had no previous preparation in differential equations.

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NOTE TO THE STUDENT

WHICH MACHINE?

There is an enormous variety of pocket calculators available with features and functions in many different combinations. Many of these calculators are suitable for learning the calculus with this book. *The recommended machine for our work is one which has buttons to calculate trig and log functions, displays at least eight digits, and has an adapter-recharger.* Of course, methods for approximating logarithmic and trigonometric functions are given in this book. Nevertheless, it is our experience that a student who attempts to do the work with a four- or seven-function machine will become distracted by the copious arithmetic and eventually will despair.

Most calculators that satisfy our minimal requirements also have square root and reciprocal functions, a memory, and the internal constant π . Most will also accept arguments for trig functions in radians as well as degrees. In addition, some such calculators have the ability to convert a result into "scientific notation,"

with a mantissa and an exponent. These machines also offer superior logical systems called either "Parenthetical" or "Reverse-Polish." These systems are very useful; they enable the machine to accept more complex formulas without the user having to rewrite equations or write down intermediate results. If you can afford it, we recommend that you use a machine having scientific notation and such a system of logic.

There are even more elaborate calculators. Some have multiple memories, and some may be programmed to perform repetitive computations automatically. Such features could be useful in our work, but they will not be necessary.

HOW TO GET STARTED

Before beginning your study of this book, you must become familiar with the machine you will be using. You can do this by reading the explanations in the owner's manual and working out the simple examples given there. This may take a few hours if it is your first calculator. You should also test your understanding by trying to do simple arithmetic on the machine, using numbers like 2, 3, and 4 and working out problems for which you already know the answer.

Some suggestions on "Invisible Registers" and simple arithmetic, as well as the evaluation of more complicated expressions, are offered in the Appendix of this book.

WHAT TO DO WHEN YOU ARE BAFFLED

You want to learn to use the calculus. You cannot do that by osmosis, by watching someone else do it, any more than you could learn to play chess, football, or the violin by close observation. Do the Exercises. They are not repetitive drills. You can expect some of the joy of discovery and creation with each solution you construct.

When you have trouble understanding a topic, reexamine the worked-out Examples and then do some Exercises. Do the easiest ones first. With experience you will come to know which you can do in your head and which are difficult for you. Calculate and write out solutions for the harder ones. You will find that you, too, can learn the calculus and enjoy doing it.