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in Mathematics  
and Its Applications**

**Volume 9**

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George R. Sell   Hans Weinberger

# Institute for Mathematics and Its Applications

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The **Institute for Mathematics and Its Applications** was established by a grant from the National Science Foundation to the University of Minnesota in 1982. The IMA seeks to encourage the development and study of fresh mathematical concepts and questions of concern to the other sciences by bringing together mathematicians and scientists from diverse fields in an atmosphere that will stimulate discussion and collaboration.

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# Hydrodynamic Behavior and Interacting Particle Systems

With 10 Illustrations



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# FOREWORD

This IMA Volume in Mathematics and its Applications

## HYDRODYNAMIC BEHAVIOR AND INTERACTING PARTICLE SYSTEMS

is in part the proceedings of a workshop which was an integral part of the 1985-86 IMA program on STOCHASTIC DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS. We are grateful to the Scientific Committee:

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Wendell Fleming  
Theodore Harris  
Pierre-Louis Lions  
Steven Orey  
George Papanicolaou

for planning and implementing an exciting and stimulating year-long program. We especially thank the Program Organizer, George Papanicolaou for organizing a workshop which brought together scientists and mathematicians in a variety of areas for a fruitful exchange of ideas.

George R. Sell

Hans Weinberger

## PREFACE

A workshop on the hydrodynamic behavior of interacting particle systems was held at the Institute for Mathematics and its Applications at the University of Minnesota during the week of March 17, 1986. Fifteen papers presented at the workshop are collected in this volume. They contain research in several different directions that are currently being pursued.

The paper of Chaikin, Dozier and Lindsay is concerned with experimental results on suspensions in regimes where modern mathematical methods could be useful. The paper of Fritz gives an introduction to these methods as does the paper of Spohn. Analytical methods currently used by in the physics and chemistry literature are presented in the paper of Freed, Wang and Douglas. The paper of Caffisch deals with time dependent effects in sedimentation.

In the papers by Ozawa, Rubinstein and Figari, Papanicolaou, Rubinstein the continuum limit of boundary value problems in regions with many small inclusions is analyzed. These are static problems but one expects that the methods used will be useful eventually for dynamic problems where the inclusions (particles) move.

The vortex method as a problem in numerical analysis is treated by Goodman and as a probabilistic one by Osada. The propagation of chaos for Burgers equation is treated by Sznitman. The papers of Dawson, Gorostiza, and Tanaka deal with probabilistic aspects of particle systems (or random media) while Loper's paper introduces a continuum mechanics model for flow of a slurry.

**George Papanicolaou**