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A.J. Chorin J.E. Marsden

A Mathematical Introduction to Fluid Mechanics

Second Edition

With 86 Illustrations

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Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM)* .

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Preface

This book is based on a one-term course in fluid mechanics originally taught in the Department of Mathematics of the University of California, Berkeley, during the spring of 1978. The goal of the course was not to provide an exhaustive account of fluid mechanics, nor to assess the engineering value of various approximation procedures. The goals were: **i** to present some of the basic ideas of fluid mechanics in a mathematically attractive manner (which does not mean "fully rigorous"); **ii** to present the physical background and motivation for some constructions that have been used in recent mathematical and numerical work on the Navier-Stokes equations and on hyperbolic systems; and **iii** to interest some of the students in this beautiful and difficult subject. This second edition has incorporated some updates and revisions, but the spirit and scope of the original book are unaltered.

The book is divided into three chapters. The first chapter contains an elementary derivation of the equations; the concept of vorticity is introduced at an early stage. The second chapter contains a discussion of potential flow, vortex motion, and boundary layers. A construction of boundary layers using vortex sheets and random walks is presented. The third chapter contains an analysis of one-dimensional gas flow from a mildly modern point of view. Weak solutions, Riemann problems, Glimm's scheme, and combustion waves are discussed.

The style is informal and no attempt is made to hide the authors' biases and interests. Moreover, references are limited and are by no means exhaustive. We list below some general references that have been useful for us and that contain fairly extensive bibliographies. References to specific points are made directly in the text.

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