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Barry C. Arnold

Majorization
and the Lorenz Order:
A Brief Introduction

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Author

Barry C. Arnold
University of California, Department of Statistics
Riverside, CA 92521, USA

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Preface

My interest in majorization was first spurred by Ingram Olkin's proclivity for finding Schur convex functions lurking in the problem section of every issue of the American Mathematical Monthly. Later my interest in income inequality led me again to try and "really" understand Hardy, Littlewood and Polya's contributions to the majorization literature. I have found the income distribution context to be quite convenient for discussion of inequality orderings. The present set of notes is designed for a one quarter course introducing majorization and the Lorenz order. The inequality principles of Dalton, especially the transfer or Robin Hood principle, are given appropriate prominence.

Initial versions of these notes were used in graduate statistics classes taught at the Colegio de Postgraduados, Chapingo, Mexico and the University of California, Riverside. I am grateful to students in these classes for their constructive critical commentaries.

My wife Carole made noble efforts to harness my free form writing and punctuation. Occasionally I was unmoved by her requests for clarification. Time will probably prove her right in these instances also. Peggy Franklin did an outstanding job of typing the manuscript, and patiently endured requests for innumerable modifications.

Barry C. Arnold
Riverside, California
July, 1986

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