Graduate Texts in Mathematics

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Preface

This book was written to make learning introductory algebraic geometry as easy as possible. It is designed for the general first- and second-year graduate student, as well as for the nonspecialist; the only prerequisites are a one-year course in algebra and a little complex analysis. There are many examples and pictures in the book. One's sense of intuition is largely built up from exposure to concrete examples, and intuition in algebraic geometry is no exception. I have also tried to avoid too much generalization. If one understands the core of an idea in a concrete setting, later generalizations become much more meaningful. There are exercises at the end of most sections so that the reader can test his understanding of the material. Some are routine, others are more challenging. Occasionally, easily established results used in the text have been made into exercises. And from time to time, proofs of topics not covered in the text are sketched and the reader is asked to fill in the details.

Chapter I is of an introductory nature. Some of the geometry of a few specific algebraic curves is worked out, using a tactical approach that might naturally be tried by one not familiar with the general methods introduced later in the book. Further examples in this chapter suggest other basic properties of curves.

In Chapter II, we look at curves more rigorously and carefully. Among other things, we determine the topology of every nonsingular plane curve in terms of the degree of its defining polynomial. This was one of the earliest accomplishments in algebraic geometry, and it supplies the initiate with a straightforward and very satisfying result.

Chapter III lays the groundwork for generalizing some of the results of plane curves to varieties of arbitrary dimension. It is essentially a chapter on commutative algebra, looked at through the eyeglasses of the geometer.

Algebraic ideas are supplied with geometric meaning, so that in a sense one obtains a “dictionary” between commutative algebra and algebraic geometry. I have put this dictionary in the form of a diagram of lattices; this approach does seem to neatly tie together a good many results and easily suggests to the reader a number of possible analogues and extensions.

Chapter IV is devoted to a study of algebraic varieties in $\mathbb{C}^n$ and $\mathbb{P}^n(\mathbb{C})$ and includes a geometric treatment of intersection multiplicity (which we use to prove Bézout’s theorem in n dimensions).

In Chapter V we look at varieties as underlying objects upon which we do mathematics. This includes evaluation of elements of the variety’s function field (that is, a study of valuation rings), a translation of the fundamental theorem of arithmetic to a nonsingular curve-theoretic setting (the classical ideal theory), some function theory on curves (a generalization of certain basic facts about functions meromorphic on the Riemann sphere), and finally the Riemann–Roch theorem on a curve (which ties in function theory on a curve with the topology of the curve).

After the reader has finished this book, he should have a foundation from which he can continue in any of several different directions—for example, to a further study of complex algebraic varieties, to complex analytic varieties, or to the scheme-theoretic treatments of algebraic geometry which have proved so fruitful.

It is a pleasure to acknowledge the help given to me by various students who have read portions of the book; I also want to thank Frank Lozier for critically reading the manuscript, and Basil Gordon for all his help in reading the galleys. Thanks are also due to Mary Blanchard for her excellent job in typing the original draft, to Mike Ludwig who did the line drawings, and to Robert Janusz who did the shaded figures. I especially wish to express my gratitude to my wife, Joan, who originally encouraged me to write this book and who was an invaluable aid in preparing the final manuscript.

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