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Preface

It is now generally recognized that the field of combinatorics has, over the past years, evolved into a fully-fledged branch of discrete mathematics whose potential with respect to computers and the natural sciences is only beginning to be realized. Still, two points seem to bother most authors: The apparent difficulty in defining the scope of combinatorics and the fact that combinatorics seems to consist of a vast variety of more or less unrelated methods and results. As to the scope of the field, there appears to be a growing consensus that combinatorics should be divided into three large parts:

(a) *Enumeration*, including generating functions, inversion, and calculus of finite differences;
(b) *Order Theory*, including finite posets and lattices, matroids, and existence results such as Hall's and Ramsey's;
(c) *Configurations*, including designs, permutation groups, and coding theory.

The present book covers most aspects of parts (a) and (b), but none of (c). The reasons for excluding (c) were twofold. First, there exist several older books on the subject, such as Ryser [1] (which I still think is the most seductive introduction to combinatorics), Hall [2], and more recent ones such as Cameron–Van Lint [1] on groups and designs, and Blake–Mullin [1] on coding theory, whereas no comprehensive book exists on (a) and (b). Second, the vast diversity of types of designs, the very complicated methods usually still needed to prove existence or non-existence, and, in general, the rapid change this subject is presently undergoing do not favor a thorough treatment at this moment. I have also omitted reference to algorithms of any kind because I feel that presently nothing more can be said in book form about this subject beyond Knuth [1], Lawler [1], and Nijenhuis-Wilf [1].

As to the second point, that of systematizing the definitions, methods, and results into something resembling a theory, the present book tries to accomplish just this, admittedly ât the expense of some of the spontaneity and ingenuity that makes combinatorics so appealing to mathematicians and non-mathematicians alike. To start with, mappings are grouped together into classes by placing various restrictions on them. To stick to the division outlined above, these classes of mappings are then counted, ordered, and arranged. The emphasis on ordering is well justified by the everyday experience of a combinatorist that most discrete structures, while perhaps lacking a simple algebraic structure, invariably admit a natural ordering. Following this program, the book is divided into three parts, the first part presenting the basic material on mappings and posets, in Chapters I and II, respectively, the second part dealing with enumeration in Chapters III to V, and the third part on the order-theoretical aspects in Chapters VI–VIII.

The arrangement of the material allows the reader to use the three parts almost independently and to combine several subsections into a course on special topics. For instance, Chapter II has been used as an introduction to finite lattices, Chapters VI and VII as a course on matroids, and parts of Chapter VII and Chapter VIII as a course on transversal theory and the major existence results. The exercises have been graded. Unmarked exercises can be solved without a great deal of effort; more difficult ones are marked with an asterisk (*). The symbol → indicates that the exercise is particularly helpful or interesting, but in no instance is the statement or the solution of an exercise necessary to the development of the subject. The references given at the end are, of course, by no means exhaustive; usually they have been included because they were used in one way or another in the preparation of the text. Books are indicated by an asterisk.

The German version of the present book appeared in two volumes—Kombinatorik I. Grundlagen und Zähltheorie; and II. Matroide und Transversaltheorie—as Springer Hochschultexts. Combining these two parts has been a more formidable task than I originally thought. Most of the material has been reorganized, with the major changes appearing in Chapter VIII due to many new results obtained in the last few years.

I had the opportunity of working as a research associate at the Department of Statistics of the University of North Carolina in the Combinatorial Year program 1968–1970. It was during this time that I first planned to write this book. Of the many people who have encouraged me since and furthered this work, I owe special thanks to G.-C. Rota, R. C. Bose, and T. A. Dowling for many hours of discussion; to H. Wielandt, H. Salzmann, and R. Baer for their constant support; to R. Weiss, G. Prins, R. H. Schulz, J. Schoene, and W. Mader, who read all or part of the manuscript; and finally to M. Barrett for her impeccable typing.

It is my hope that I have been able to record some of the many important changes that combinatorics has undergone in recent years while retaining its origins as an intuitively appealing mathematical pleasure.

Berlin M. Aigner
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